ScienceWord and PagePlayer
Graphical representation

Dr Emile C. B. COMLAN

Novoasoft Representative in Africa

Emails:

ecomlan@scienceoffice.com

ecomlan@yahoo.com

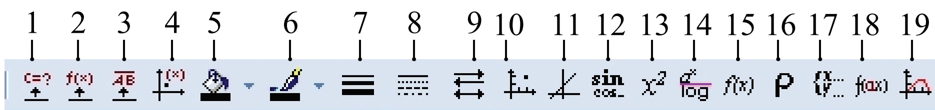
Web site: www.scienceoffice.com

Graphical representation

1) General view

ScienceWord has mathematical tools for graphical representation of real functions, parametrical curves, curves in polar coordinates, etc.

Click on the "**Coordinate System** " button in the geometry toolbar. Then, click on the worksheet and hold down the left button of the mouse and drag it to get the desired axes sizes. The tools shown below appear automatically in the geometry toolbar



To find out the function of a tool, just place the pointer on it. After a few seconds, this function is displayed.

1- Define an independent variable; 2- Functional variable; 3- Define a vector; 4- Define a point coordinates; 5- Fill color; 6- Brush color; 7-Line width; 8- Line style; 9- Arrow; 10- Create a coordinates system point; 11- Draw straight line; 12- Graph trigonometric function; 13- Graph conics; 14- Graph exponential and logarithmic functions; 15- Graph function in coordinates system; 16- Polar curve; 17- Graph parametric function; 18- Multi parameters function; 19- Create broken line with given data.

The following table shows the elementary functions list available

Functions list		
Symbol	Name	Meaning
+ − * /	Addition - Substraction - Multiplication − Division	$(3*x-5)/2+1=\frac{3x-5}{2}+1$
pi	pi	pi=π
^	Power	$x\hat{}3=x^3$
%	Remainder	5%3=2；5.4%2.6=0.2
abs(x)	Absolute value	abs(x)=|x|
sin(x)	Sine	sin(x)=sinx
cos(x)	Cosine	cos(x)=cosx
tan(x)	Tangent	tan(x)=tanx
asin(x)	Sine inverse	asin(x)=arcsinx
acos(x)	Cosine inverse	acos(x)=arcosx
atan(x)	Tangent inverse	atan(x)=arctan*x* Values domain : [-pi/2, pi/2]

atan2 (y, x)	(x,y) point polar angle in radians	Values domain: [- pi, pi] If y< 0 , then atan2 (y, x) < 0 If y > 0, atan2 (y, x) > 0
cosh (x)	Hyperbolic cosine	cosh (x) = cosh$x$
sinh (x)	Hyperbolic sine	sinh (x) = sinh$x$
tanh (x)	Hyperbolic tangent	tanh (x) = tanh$x$
ceil(x)	Ceil function	If n⩽x< n+1, ceil(x)=n+1, $n \in \mathbb{Z}$
deg(x)	Degree function	deg(x) = $\frac{180\,x}{\pi}$
exp(x)	Exponential function	exp (x) = e^x
floor (x)	Floor function	If n⩽x< n+1, floor (x)=n, $n \in \mathbb{Z}$
hypot(x,y)	Hypothenuse function	hypot (x,y) = $\sqrt{x^2+y^2}$
inint (x,r0,r1)	Inverse function (Ran(x)) $^{-1}$	inint (f (x) , 2,6)=f^{-1}[2,6]
max (x,y)	Max function	max (3,9) = 9
min(x,y)	Min function	min (3,9) =3
mod(x,y)	Modulo function	mod(x,y) = x%y
ln (x) 、 log(x)	Neperian Logarithm	ln (x) = log (x) = $\log_e$ (x)
log10 (x)	base 10 logarithm	log10 (x) = $\log_{10}$ (x)
pow (x,y)	x power y	pow (x. y) = x^y
rad(x)	Radian function	rad(x)= $\frac{x}{180}\pi$
sign(x)	Sign function	If x> 0, sign(x) = 1 If x=0, sign(x) = 0 If x<0, sign(x) = -1
step (x)	Step function	If x≥0, step (x) = 1 If x<0, step (x) = 0
Sqrt (x)	Square root function	Sqrt (x)= $\sqrt{x}$
j0 (x) , j1(x) , jn(x)	First function of Bessel: degree 0, degree 1, degree n	j0 (x) = J(0, x) j1(x) = J (1, x) jn(x)=J(n. x)
y0(x) , y1(x) , yn(n,x)	Second function of Bessel: degree 0, degree 1, degree n	y0(x)=Y (0. x) y1(x)=Y (1, x) yn(x) = Y (x, n)

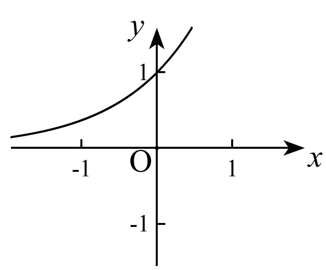
<u>Notes</u>

- In trigonometric function or Polar coordinates system dialog box, when the "Angle in pi Radian" box is ticked off (☑ Angle in Radian) , then the values of the "*Domain*" are displayed in π radians . To get the configuration of "*Domain*" in numerical values, uncheck the "Angle in pi Radian" box.
- The position of a point or a line in the coordinate system can be modified with the mouse or through their properties.
- The variable under consideration in the graphical representation of a function does not always remain the same. There exist four types of variable: x, y, t and r. For example, an expression such as $y = f(x)$ shows that the accepted variable is x; an expression such as $t = f(r)$ shows that the accepted variable is r; etc.

2) Two dimensional common graphical representation

We are going to represent the function: $y = e^x$

Click on the "***Coordinate system***" button in the drawing toolbar, while the pointer turns into a pencil " ", hold down the left button of the mouse and then dragging the mouse, create coordinate system of the desired size. From the tools that appear, click on "f(x) Graph Function in Coordinate System". In the dialogue box opens up, write the appropriate expression of y, that is e^x or exp (x) . Then Click on "OK" to get the curve as follows

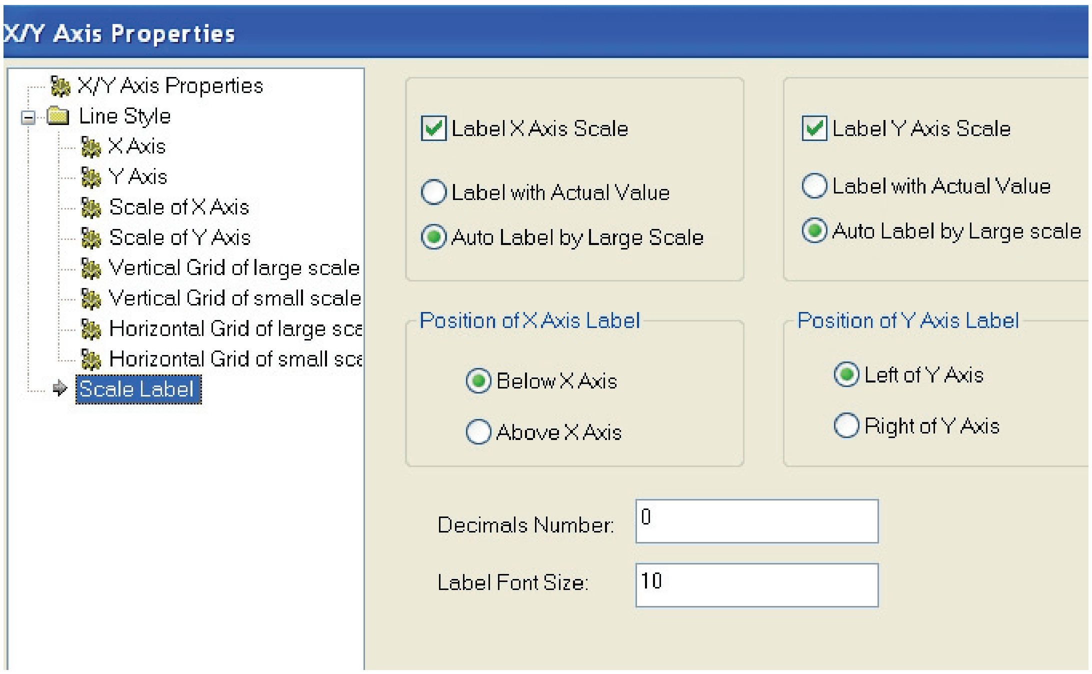


If you want to make some modifications on the curve, then double left- click with the mouse on the curve to open its "***Properties***" dialog box

If you want to make some modifications to the axes of the coordinates, just double click on any axis to open axis X/Y properties dialog box. Then you can carry out the desired modifications.

Note:

In X/Y axes properties dialog box you can click on Scale label to get the following dialog box where you can set up Label axis scale, Label position, the number of decimals of Label and Label font size.



For example, if the large scale increment (in X/Y axis properties dialog box) of X axis is 15 and the large scale increment of Y axis is 125, you may have to check "Label with actual Value" in Label X axis scale and Label Y axis scale options to get Fig1 as axes coordinates. But with the auto label option you will get Fig2.

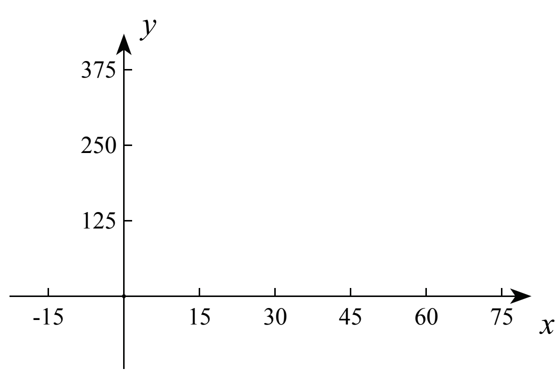


Fig1

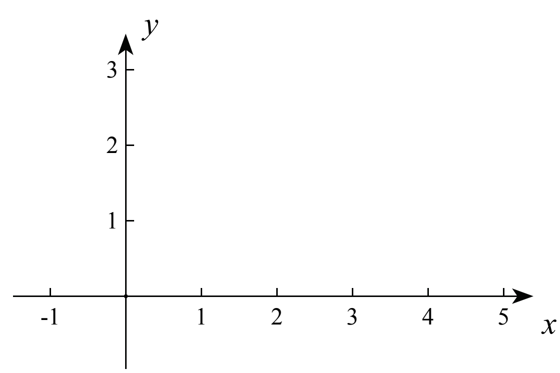


Fig2

Then for any function graph to be plotted in Fig1, you may have to consider in the function dialog box the following domain: xs = -15 and xe = 75 .

Then for any function graph to be plotted in Fig2, you may have to consider in the function dialog box the following domain: xs = -1 and xe = 5

Note

You can draw in axes coordinates system many function curves of all types.
When a line, a circle, an ellipse in axes coordinates system, their equations are displayed in object properties dialog box

You can define variables and use them as parameters in multi parameter function  dialog box to create animated function curve.

3) Principles of graphical representation

Let's remind that elementary functions refer to: polynomial, rational, trigonometric, power, exponential, logarithmic and absolute value functions.

a) Graphical representation and domains

Given an elementary function of type $y=f(x)$ or $x=f(y)$ and given D_f its domain of definition.

- If $D_f=\mathbb{R}$, the graphical representation is directly done. This for example as in the case of an exponential function.

- Let's suppose that $D_f = \mathrm{I}_1 \cup \mathrm{I}_2 \cup \ldots \cup \mathrm{I}_n$, $n \geqq 2$, and I_1, I_2, ..., I_n are intervals with any two intervals empty intersection, I_1 having the smallest lower bound and I_n having the highest upper bound.

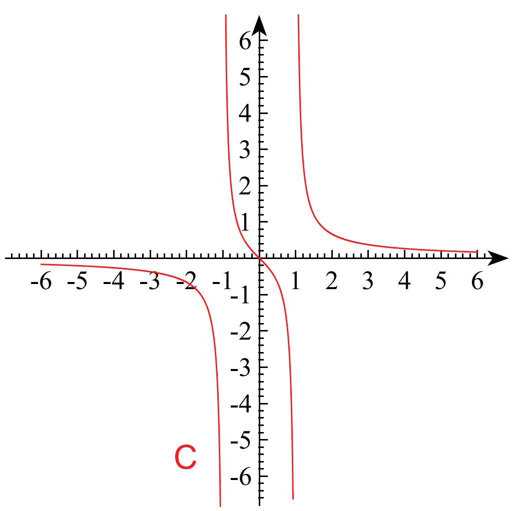
* If I_1 is a left-closed interval having a as the lower bound and I_n is a right -closed interval having b as the upper bound, then the graphical representation can be done directly on the right -closed interval I of bounds a and b (a and b are finite or infinite) .

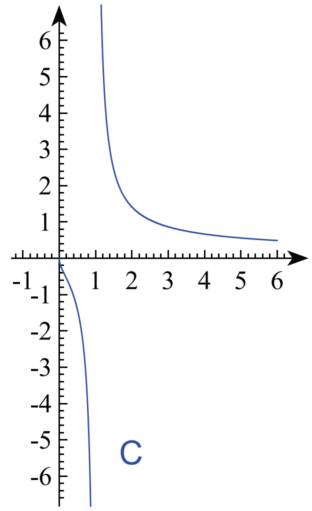
For example, let's consider the following functions: $f(x)=\dfrac{x}{x^2-1}$ and $g(x) = \dfrac{\sqrt{x}}{x-1}$.

Then $D_f =]-\infty,-1[\cup]-1,1[\cup]1,+\infty[$ and $D_g = [0,1[\cup]1,+\infty[$..

The graphical representation of f and g can be respectively carried out directly on $\mathbb{R}$ set and $[0,+\infty[$ interval.

In a practical way, to get the curves shape of these functions, you just need to represent respectively f on $]-6,6[$ and g on $[0,6]$. We have the following results:





* If I_1 or I_n do not meet the previous conditions, then the graphical representation will require that the variable be indeed chosen on a part of the function domain D_f.

* If D_f is a segment, the graphical representation also requires that the variable be indeed chosen on a part of that segment.

b) Note on function power

In ScienceWord as it is the case of most of the scientific software, the function power $x \mapsto \sqrt[n]{x}$ is considered as being defined on $[\,0, +\infty\,[$.

Given that for all odd values of n, the function power is defined on $\mathbb{R}$, so the graphical representation of $y = \sqrt[n]{x}$ in ScienceWord (in case n is odd) , use anyone of the three following methods.

i) Given the fact that $y = f(x) = \sqrt[n]{x}$ on $[\,0, +\infty\,[$ and $y = -\sqrt[n]{-x}$ on $[\,0, -\infty\,[$, then we can just consider the graphical representation of $f(x) = \text{sign}(x)\,\sqrt[n]{\text{sign}(x)\,x}$.

ii) We represent $x = g(y) = y^n$ on $\mathbb{R}$.

iii) We represent parametrical function defined as $\begin{cases} x = t^n \\ y = t \end{cases}$, $t \in \mathbb{R}$.

In general, to represent directly the function defined by $y = \sqrt[n]{g(x)}$, where n is an odd integer, it is suitable to just consider its expression $y = \text{sign}(g(x))\,\sqrt[n]{\text{sign}(g(x))}$.

For example, $y = \sqrt[3]{x+1} - \sqrt[5]{x^3+x^2+1} + 2 = \text{sign}(x+1)\,\sqrt[3]{\text{sign}(x+1)(x+1)}$

$-\text{sign}(x^3+x^2+1)\,\sqrt[5]{\text{sign}(x^3+x^2+1)(x^3+x^2+1)} + 2.$

c) Note on conics equations in 2D coordinates system

In ScienceWord and PagePlayer, a conic equatiosn is considered as follow

$\begin{cases} G(x, y)=Ax^2+Cy^2+Dx+Ey+F=0 \\ \theta=\theta_0 \end{cases}$, where in the conic properties dialogue box α is the angle between abscissa x axis and conic a axis; $G(x, y)=0$ is the equation of that conic in the coordinates axes obtained after a rotation of xy axes about the origin of θ_0. .

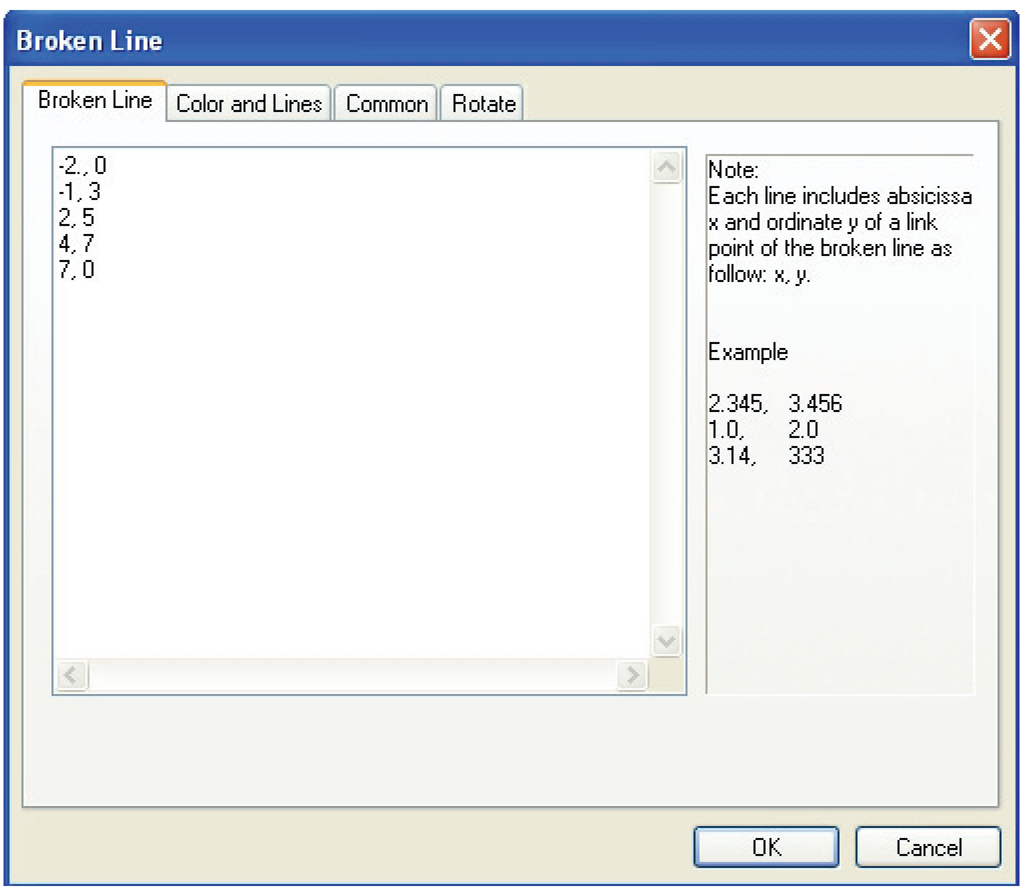
In general, the equation of the conic in the original coordinates system is given as follow: $G(x\cos\theta_0+y\sin\theta_0, -x\sin\theta_0+y\cos\theta_0)=0$. Then,

if $\theta_0=0$ (the default settings) , this equation is just $Ax^2+Cy^2+Dx+Ey+F=0$;

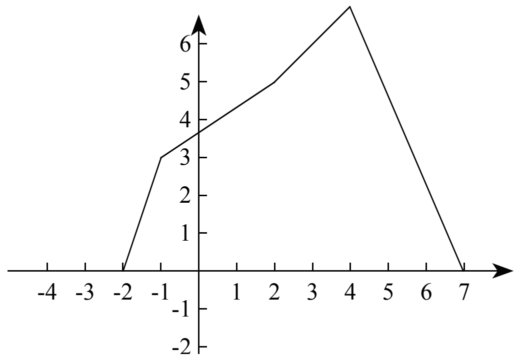
if $\theta_0\neq0$, this equation is: $A(x\cos\theta_0+y\sin\theta_0)^2+C(-x\sin\theta_0+y\cos\theta_0)^2+D(x\cos\theta_0+y\sin\theta_0)+E(-x\sin\theta_0+y\cos\theta_0)+F=0$

4) Joining points in the coordinate system

Click on tool that appears when the coordinates system is selected. In the dialog box that appears, enter data as shown below.



Then click on "OK" button. The following result is obtained:



Note that you can copy data of the worksheet and paste them directly in the "Broken Line" dialog box and vice versa.

5) Image of a (C) graph by plane isometry

a) Method used

Here the technique used is based on parametrical equations where for any point M (x, y) of (C) , we have:

$$\begin{cases} x = h(t) \\ y = g(t) \end{cases} t \in E \quad (E \text{ being a subset of } \mathbb{R}) .$$

For example, if (C) is the graph of real function $y = f(x)$, with $x \in D_f$, then we consider the parametrical equations:

$$\begin{cases} x = t \\ y = f(t) \end{cases}, \ t \in D_f \quad ①$$

- If (C) is the graph of real function $x = f(y)$, with $y \quad D_f$, then we consider the parametrical equations:

$$\begin{cases} x = f(t) \\ y = t \end{cases}, \ t \in D_f \quad (1')$$

- If (C) is the graph of polar equation $r = f(t)$, with $t \ D$, then we consider the parametrical equations:

$$\begin{cases} x = f(t) \cos t \\ y = f(t) \sin t \end{cases}, \ t \in D \quad ②$$

If (C) is the graph of polar equation $t = f(r)$, with $t \in D$, then we consider the parametrical equations:

$$\begin{cases} x = t\cos\,(f(t)) \\ y = t\sin\,(f(t)) \end{cases},\; t \in D \quad \textcircled{2'}$$

b) Analytical expression of plane isometrics

Let consider a plane point M$(x,\ y)$ and an isometry I and let suppose that M′ (x', y') is the point of plane such as $I(\text{M}) = \text{M}'$.

- If I is the translation with respect to the vector $\vec{u}\begin{pmatrix} a \\ b \end{pmatrix}$, then $\begin{cases} x' = x + a \\ y' = y + b \end{cases}$.

- If I is the central symmetry about the centre $\Omega(x_0, y_0)$, then

$$\begin{cases} x' = -x + 2x_0 \\ y' = -y + 2y_0 \end{cases}.$$

- If I is the symmetry with respect to the axis $y = ax + b$, then

$$\begin{cases} x' = \dfrac{2a}{a^2+1}y - \dfrac{a^2-1}{a^2+1}x - \dfrac{2ab}{a^2+1} \\[2ex] y' = \dfrac{a^2-1}{a^2+1}y + \dfrac{2a}{a^2+1}x + \dfrac{2b}{a^2+1} \end{cases}.$$

- If I is the symmetry with respect to the axis $x = b$, then $\begin{cases} x' = 2b - x \\ y' = y \end{cases}$.

- If I is the rotation about the centre $\Omega(x_0,\ y_0)$ and angle θ, then

$$\begin{cases} x' = (x - x_0)\cos\theta - (y - y_0)\sin\theta + x_0 \\ y' = (x - x_0)\sin\theta + (y - y_0)\cos\theta + y_0 \end{cases}$$

Note: *you can rotate any graph about the axes origin directly through "Rotate" option of this graph properties.*

c) Image of the graph C

Let denote C $= I($C$)$ where I is an isometry .

Assume that the parametrical equations of C are: $\begin{cases} x = h(t) \\ y = g(t) \end{cases}$.

When replacing in isometry I analytical expression x by $h(t)$ and y by $g(t)$, you would obtain an expression like: $\begin{cases} x' = H(t) \\ y' = G(t) \end{cases}$.

Then , it follows that parametrical equations of C are $\begin{cases} x = H(t) \\ y = G(t) \end{cases}$.

<u>Application Example</u>

Let C be the graph of function $y = f(x)$, which parametrical equations are: $\begin{cases} x = t \\ y = f(t) \end{cases}$.

Assume that I is the symmetry with respect to the axis $y = x$, and having parametrical equations $\begin{cases} x' = y \\ y' = x \end{cases}$.

When replacing x by t and y by $f(t)$, we have: $\begin{cases} x' = f(t) \\ y' = t \end{cases}$.

Thus, the parametrical equations of C $= I$(C) are: $\begin{cases} x = f(t) \\ y = t \end{cases}$ (i).

<u>Remark:</u>

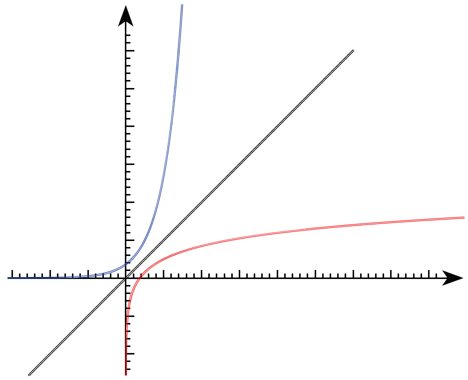
You could also consider Cartesian equation of C when of such an equation is easy to find out. For example, it is easy to get from (i) the Cartesian equation of C, that is $x = f(y)$ (ii).

In ScienceWord, you can complete C through (i) or (ii).

Assume that C is the graph of function $y = e^{2x-1}$. Then the parametrical equations of C are given as follows: $\begin{cases} x = e^{2t-1} \\ y = t \end{cases}$ $t \in \mathbb{R}$.

It is easy to notice that the Cartesian equation of C is $x = e^{2y-1}$.

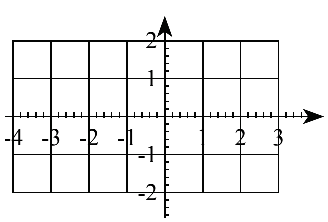
The curves (C) and (C) are completed below

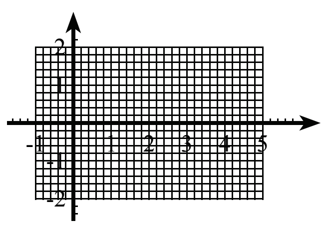


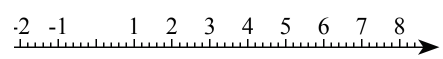
6) Flexibility of ScienceWord coordinate system

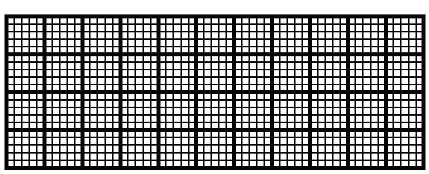
Let mention that the coordinate system is very flexible.

- You can shift the origin of the coordinate system. To proceed, click on the origin and whilst the pointer turns into the shape , hold down the left button of the mouse and move gently the origin.
- Thanks to many options in axes properties, you could get different kinds of configurations of the coordinate system as shown in the following.









7) Drawings and coordinates system

When the coordinates system is selected, any geometrical object (line, rectangle, free curve or Bezier curve, etc.) or any experimental tool of chemistry, physics, optics, mechanics, electromagnetism drawn in the active zone of the selection of the coordinates system, is automatically captured. It thus becomes an element of the coordinates system! When these objects are not drawn in the active zone of the coordinates system, you can merge these objects with the coordinate using the "Combine" tool.

Furthermore you can determine the coordinates of any point of this object in the coordinates system! So many practical applications! Without going to the point of drawing an exhaustive list, let just mention that it is henceforth possible to measure rapidly the level of a liquid in a "U Tube", to obtain the coordinates of any point in a geometrical transformation, the impact of a bullet in a shooting experience, an accurate estimate of laboratory experiment results, interesting approximate of solutions of several algebra equations, etc.

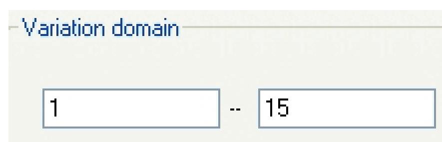
8) Filling and functions curves intersection

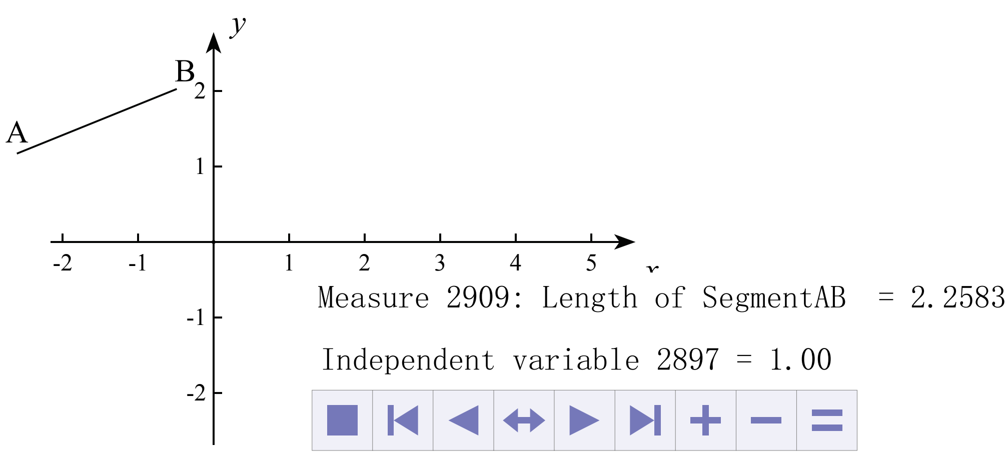
When function curves and geometrical objects are selected, the " Select and Fill Region" tool that appears in the Geometry Toolbar task zone helps to fill their intersection or their difference or their union region.

When two function curves or a curve and a line are selected the " " Intersection of two curves" tool that appears in the Geometry Toolbar task zone helps to draw the intersection points.

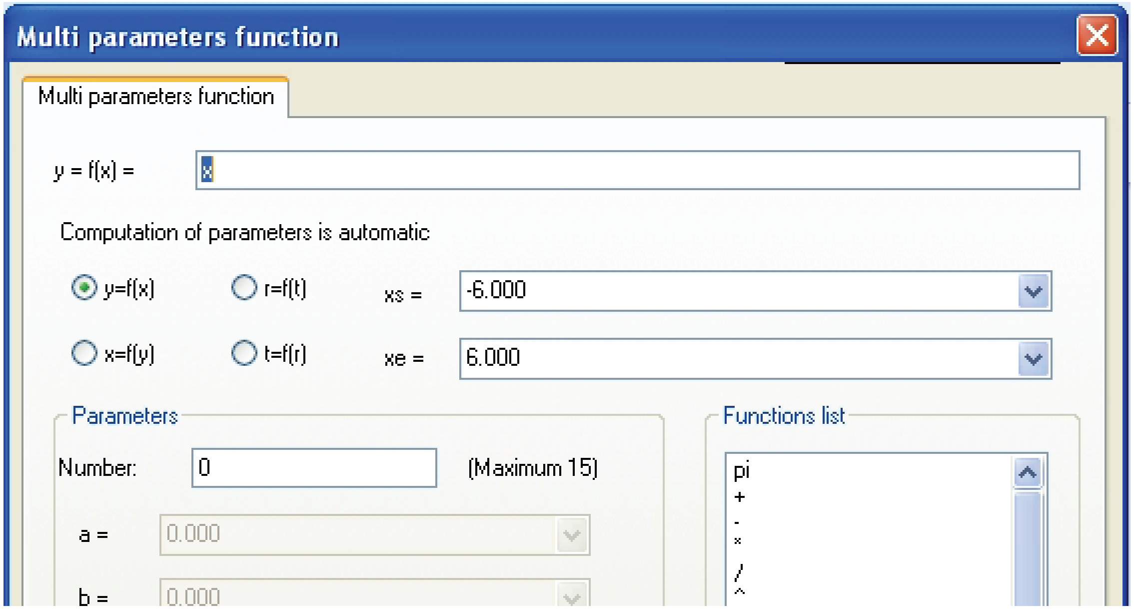
9) Graph animation

a) Using the multi parameter function

Draw in the coordinate System a segment AB and click on the icon "Length " that becomes automatically available in geometry toolbar, to display its length. Then define an independent variable with variation domain 1 to 15 .



Click on "Multi parameter function " tool to open the following dialog box

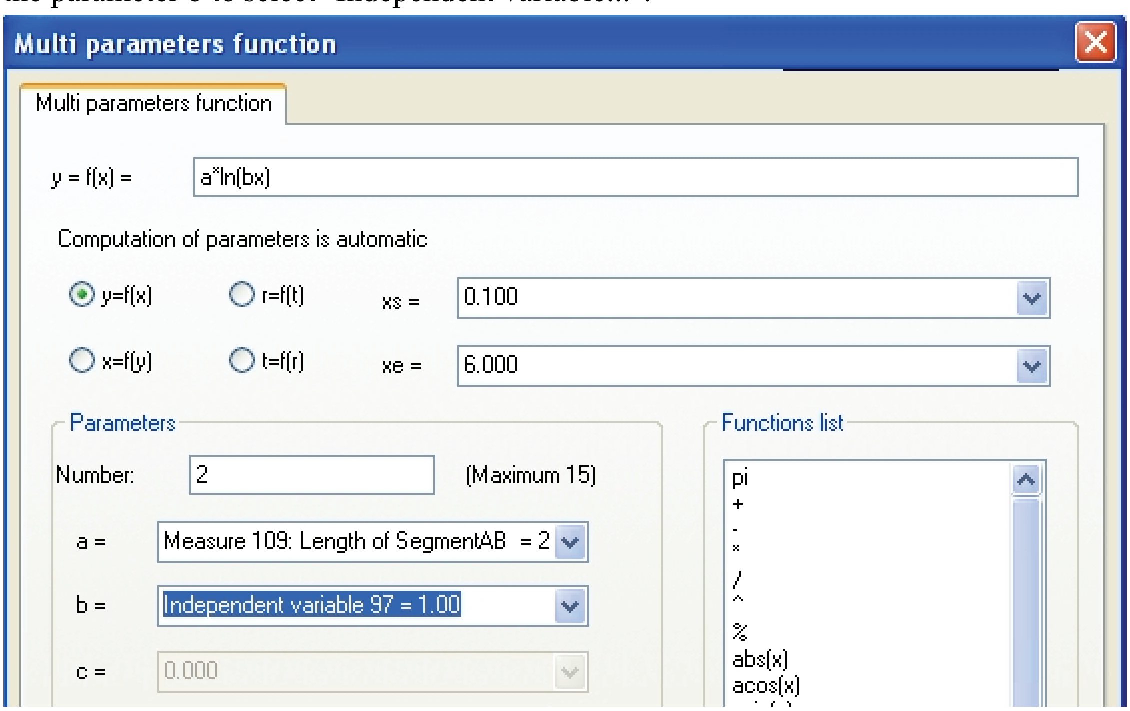


Now we are to plot the parameter function f (x) = aln (bx) where a is the length of AB segment and b the independent variable..

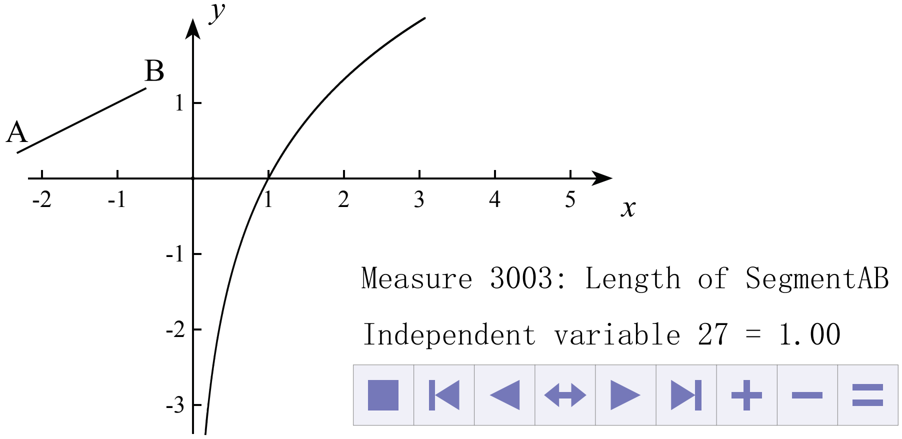
Type in y=f (x) window the expression: a∗ln (b∗x) .

As ln function is defined on] 0, +∞ [, in the domain bounded values, replace the value -6 of xs by 0.1.

In the parameter Number window replace the value 0 with the value 2. . Then the parameters a and b windows are automatically activated. Click on the drop-down button of the parameter a to select "Length of Segment AB..."; Click on the drop-down button of the parameter b to select "Independent variable...".



Then click OK button to get the graph

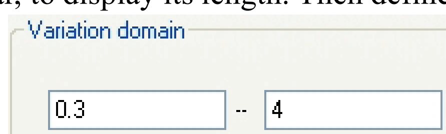


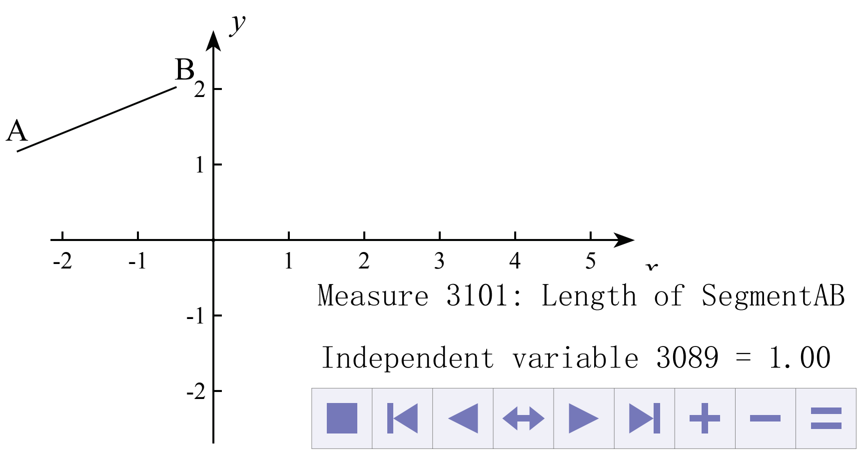
Now select the point A and move it. As the point is changing position, its length is varying and then the graph also change accordingly to the length of AB.
Animate the independent variable. Then the graph change accordingly to the value of the independent variable.

:**Note**:: The started value xs and the end value xe of the domain are also variable. You may define them as variables. The drop-down button help to select any defined variable.

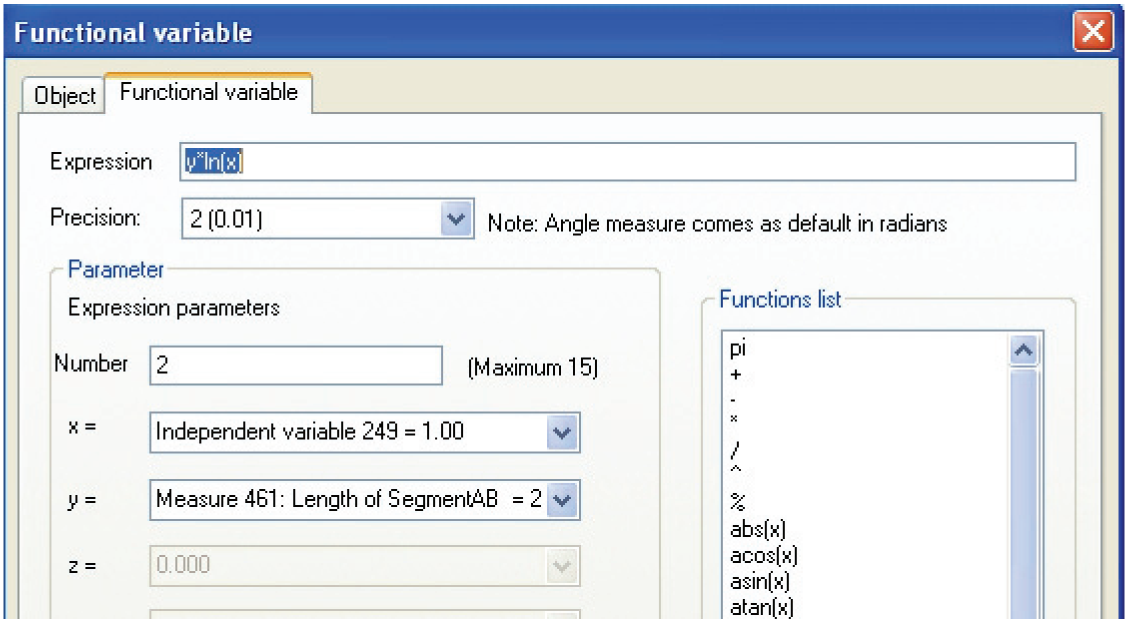
b) Using the " define point coordinates" tool

Draw in the coordinate System a segment AB and click on the icon "Length " that becomes automatically available in geometry toolbar, to display its length. Then define an independent variable with variation domain 0.3 to 4

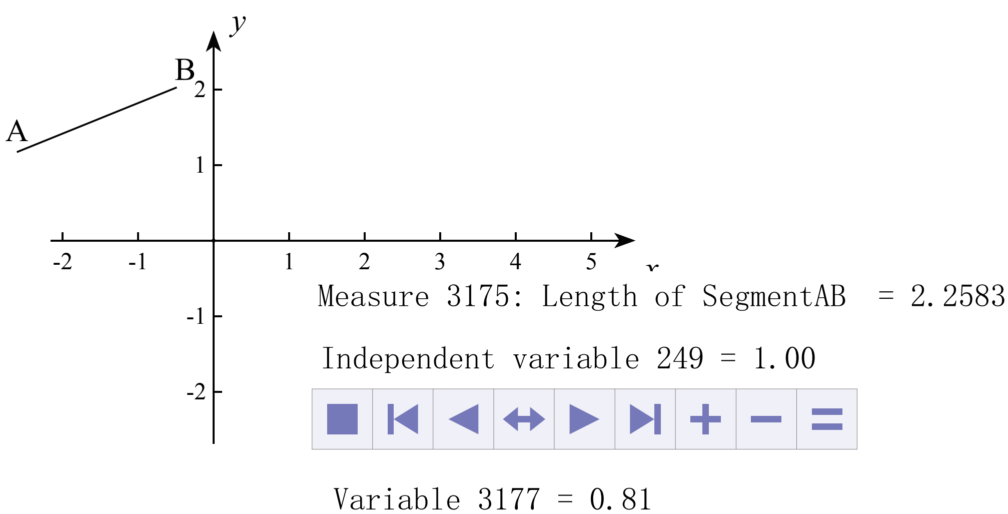




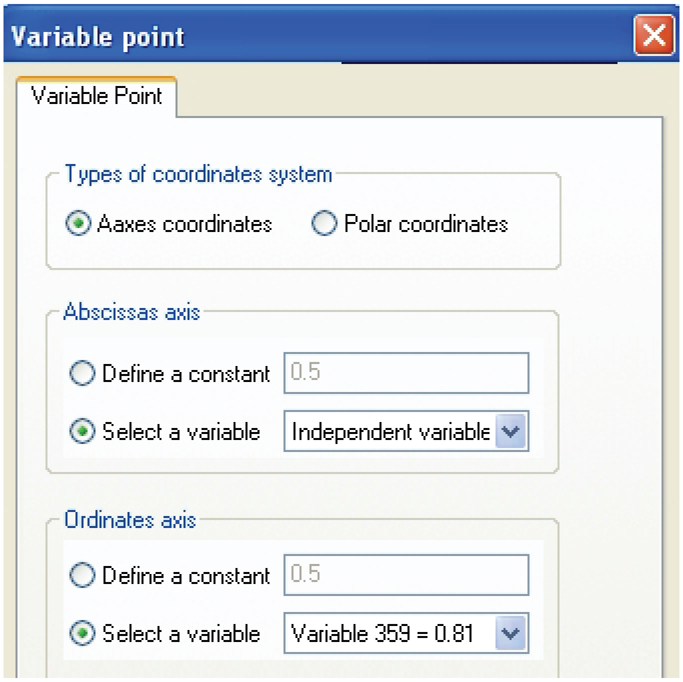
Now click on " functional variable" tool and type in the expression box: y∗ln (x)



Click OK to get a functional variable defined (in the following figure you can see variable 359) .

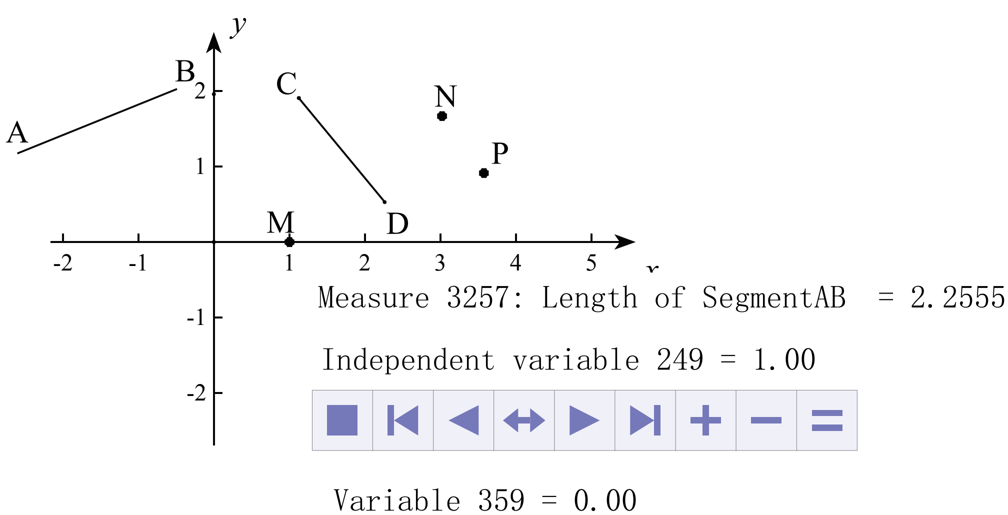


Now click on **define point coordinates.** In the dialog box that opens up consider "Axes coordinates" option and check "Select a variable" option for each axis. Then use the drop-down button to select "Independent variable" as abscissa and "Variable 359" (functional variable) as ordinate.

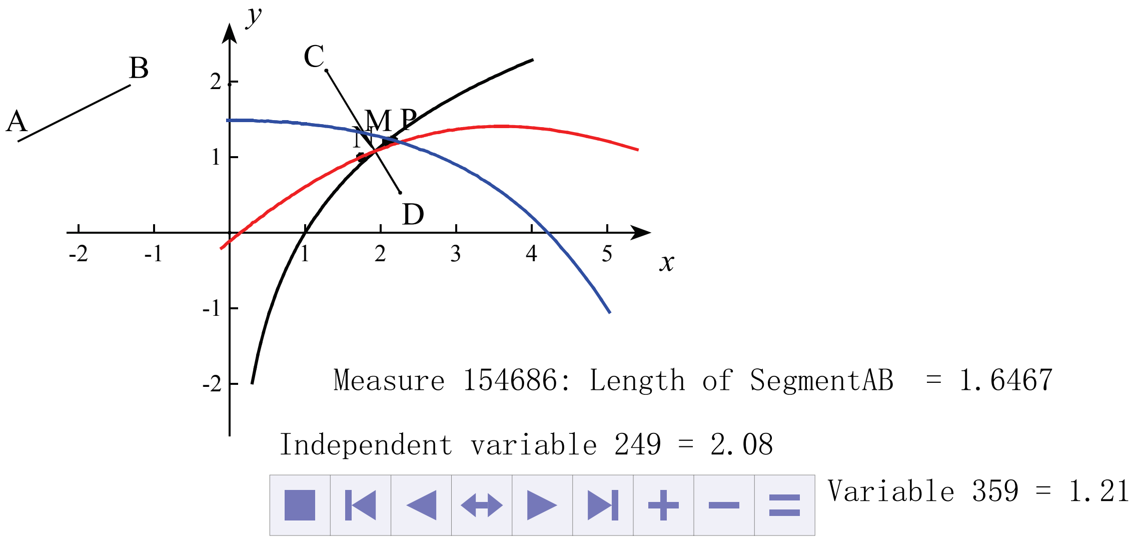


Click OK to get the variable point M having the independent as abscissa and the functional variable as ordinate.
Now animate the independent variable and note that the variable point describe the graph defined by $y = \left\| \overrightarrow{AB} \right\| \ln(x)$.

We have to generate this curve. But first let draw another segment CD and the symmetric point N of M across (CD). Then rotate -30° the point N about D to get point P..



Now select in this order the independent variable and the point M. and click on " Display the track…." that becomes available in geometry toolbar. In the dialog box that opens up replace the number 30 with 100. Click OK. Then you will get the black curve passing M point as shown as follows.



To get the symmetric across (CD) of this curve, select in this order the independent variable and the point N. and click on " Display the track....", The result is the red curve.

To get the rotation -30° of the red curve about the point D, select in this order the independent variable and the point P. and click on " Display the track....", The result is the blue curve.

Now the point A of the segment AB and move it.. You can note that the three curves follow up as the length of segment AB is a variable factor.

Animate the independent variable to see the change of position of M,.N and P

Notes

- You can bring modifications to these curves through their **properties** dialog box and through their **motion properties** dialog box.

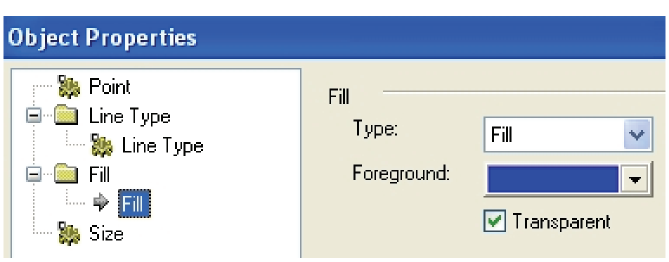


Fig 1

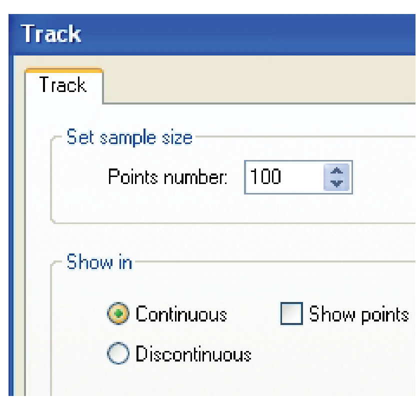


Fig 2

Insted of considering a length of the segment AB as variable factor, you may consider other independent variable or functional variable or an area, etc.

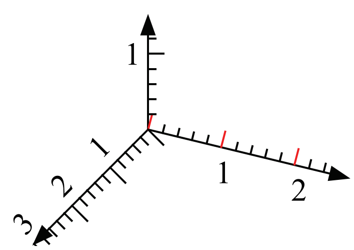
- . For the purpose of a particular animation it may be suitable to use the following kind of functions where a is a real constant.

$$h(x)=\begin{cases} f(x), if\ x<a \\ g(x), if\ x\geqslant a \end{cases} \quad , \quad k(x)=\begin{cases} f(x), if\ x\leqslant a \\ g(x), if\ x>a \end{cases}$$

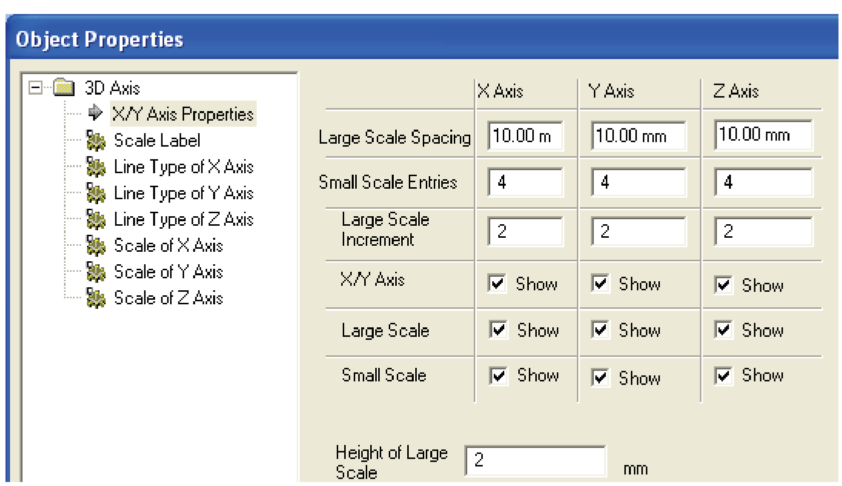
When we consider the following functions
$q(x)=\text{sign}(1+\text{sign}(x-a))$ and $p(x)=\text{sign}(1-\text{sign}(x-a))$,
then we can write:
$h(x)=g(x)q(x)+f(x)(1-q(x))$ and $k(x)=f(x)p(x)+g(x)(1-p(x))$
.

10) Three Dimension graphical representation

Click on **"3D Coordinates "** in the Geometry Toolbar. The pointer changing to the shape of a cross (+) on the worksheet, click and hold the left button of the mouse, then drag it slightly to draw the coordinates system as shown in the following illustration:

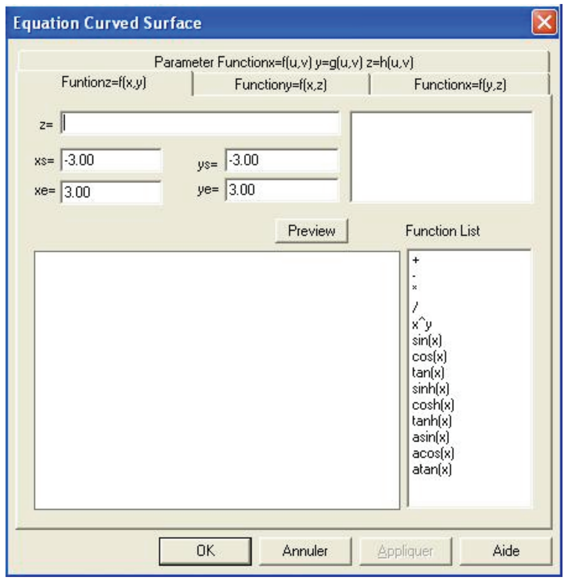


You can access the following "Object Properties" dialog box of the coordinates system by clicking on "Properties" from the contextual menu.



The above dialog box helps to carry out modifications to the coordinates system.

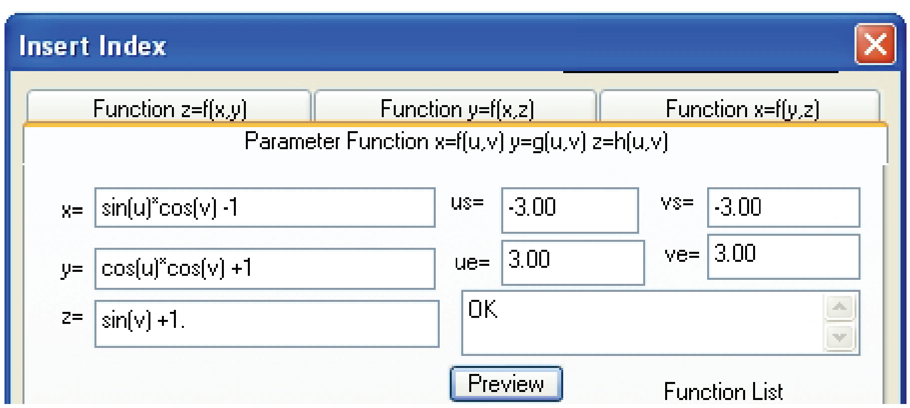
While the coordinates system is active (that is selected) , click on " z= f(x,y) Create 3D graphics" tool in the Geometry Toolbar. The following dialogue box appears:



Taking into account the list of accepted elementary functions and the definition domain, you can as it is the case in two dimensional coordinates system, plot graphs in three dimensional coordinates system.

As the case in 2 D, the coordinates system 3 D is very flexible. You can in the same way displace the origin of the coordinates system by the mouse.

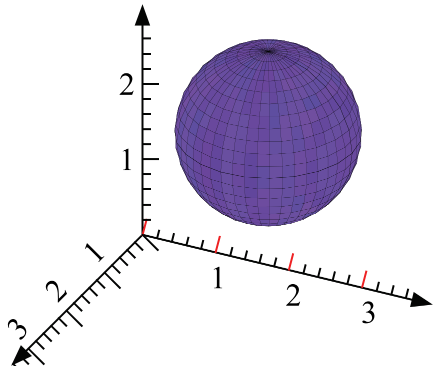
For example, to represent the sphere with radius $r = 1$ and centre (-1, 1, 1) , click on the "Parameter Function.." button, and then set the dialogue box as follows.



x = sin (u) *cos (v) -1, y = cos (u) *cos (v) +1, z = sin (v) +1.

u_s= -3.14 , u_e= 3.14, v_s= -3.14, v_e= 3.14.

Click on "OK" button to get the sphere as shown next.



It is possible to sketch in the same coordinates system many surfaces as shown below

A half of sphere defined as

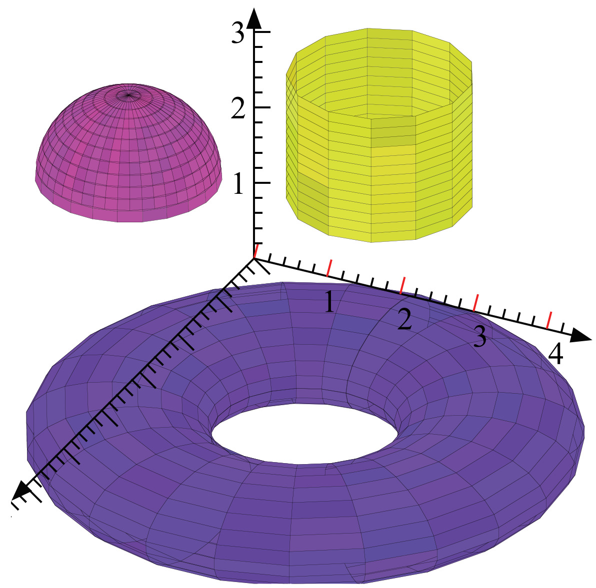
$$\begin{cases} x = \sin(u)\cos(v) - 1 \\ y = \cos(u)\cos(\text{v}) - 2 \quad -3.14 \le u \le 3.14\ ;\ 0 \le v \le 3 \\ z = \sin(v) \end{cases}$$

A cylinder defined as

$$\begin{cases} x = \sin(u) + 3 \\ y = \cos(u) + 3 \qquad -3.14 \le u \le 3.14\ ;\ \ 0 \le v \le 1.5 \\ z = v + 3 \end{cases}$$

A torus defined as

$$\begin{cases} x = (3 + \cos(v))\cos(u) \\ y = (3 + \cos(v))\sin(u) + 0.6 \quad -3.14 \le u,\ v \le 3.14 \\ z = 0.5\sin(v) - 2 \end{cases}$$



CONTENTS

Graphical representation... 2

1) General view... 2

2) Two dimensional common graphical representation.. 4

3) Principles of graphical representation... 6

a) Graphical representation and domains.. 6

b) Note on function power.. 7

c) Note on conics equations in 2D coordinates system.. 8

4) Joining points in the coordinate system.. 8

5) Image of a (C) graph by plane isometry.. 9

a) Method used... 9

b) Analytical expression of plane isometrics... 10

c) Image of the graph C... 10

6) Flexibility of ScienceWord coordinate system.. 12

7) Drawings and coordinates system.. 12

8) Filling and functions curves intersection.. 13

9) Graph animation.. 13

a) Using the multi parameter function... 13

b) Using the " define point coordinates" tool... 15

10) Three Dimension graphical representation... 19

CONTENTS.. 22