Graphical representation in ScienceWord and Class

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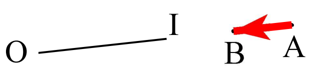
I) Line coordinate

Click on a line tool and draw a segment OI, where O is the starting point and I the end point.

The segment OI can be taken as a line as well as an axis vector unit while O is the origin and OI the unit of length.
When the segment OI is selected, you can draw any point M of the line (OI) upon clicking on " Select a point of a line " tool. Then when the point M of the line (OI) is drawn and selected, you can get its abscissa in (O, I) coordinate system upon clicking on the tool " =Ratio of directed distances" automatically available in drawing toolbar.

To plot a point P given its coordinate p in (O, I) coordinate system, just select the segment OI and click on the tool " Define an axis point abscissa".

Note 1
Any couple of points (A, B) can be chosen as a new coordinate system on the line (OI) . In fact, you just need to draw the segment AB where A is the starting point and B the ending point..



- To plot a point Q on (OI) given its coordinate q in (A, B) system, you just need to select AB and use the " Define an axis point abscissa".tool automatically available in geometry toolbar.
- Reversly, when you select AB and click on " Select a point of a line " tool to draw a point of Q of the line (OI) , the " =Ratio⋯ " tool that pops up in geometry toolbar helps to display the abscissa q of the point Q in (A, B) system.

Note 2
In general, if M is a point of the line (OI) , then the abscissa m of M in (A, B) coordinate system is given as follows:

$m = \frac{AM}{AB}\cos(\angle BAM)$ (where $\mathbf{\cos(\angle BAM)}$ is found with a precision of 0 digit) .

The functional variable tool is helpful for the calculation of m and this knowledge proves to be very useful for dynamic constructions.

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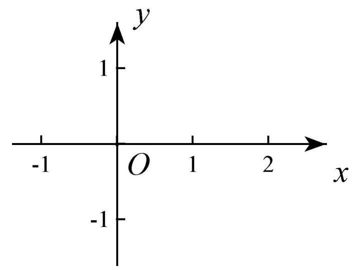
II) 2D coordinates system

ScienceWord and Class have mathematical tools for graphical representation of real functions, parametrical curves, curves in polar coordinates, etc.

Click on the "**Coordinate System** " button in the geometry toolbar. Then, Then, while the pointer of the mouse turns into a plus " + " on the worksheet, hold down the left button of the mouse and drag it to get the desired axes sizes .
You can select the label O and set its format through properties dialog box or delete it.



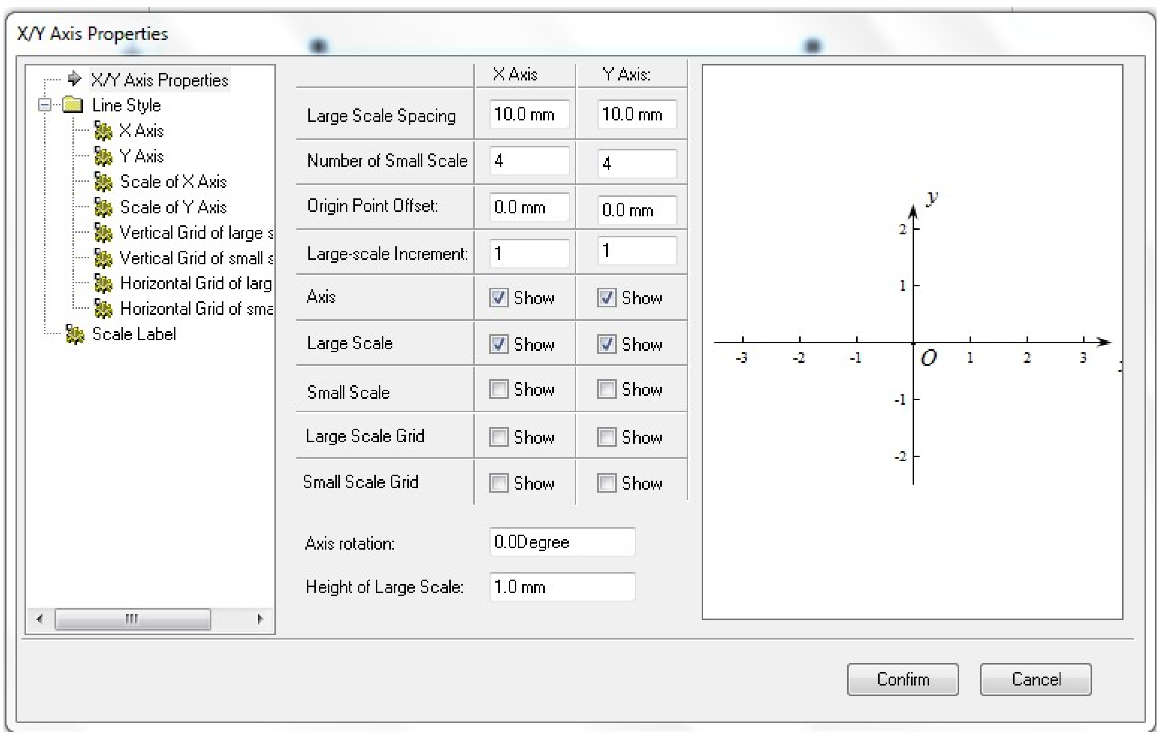
1) Axes properties dialog box

To access the axes properties dialog box, use one of the following methods

Method1: Just click on any axis (x axis or y axis) , then right click. In the contextual menu that opens up, click on "Properties".

Method2: Just double-click on any axis to access directly the axes properties box, .

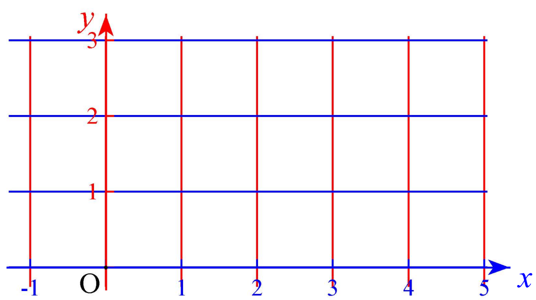
These axes properties box is shown as follow

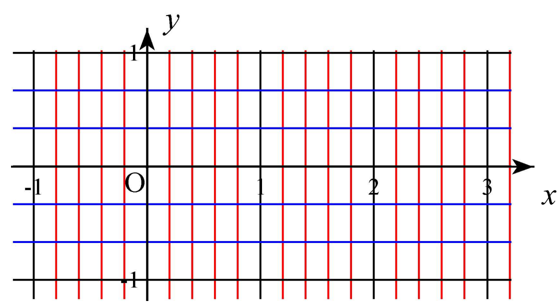


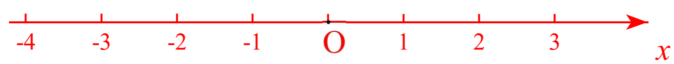
To set the grid, just check the "show" box of "Large Scale Grid" or "Small Scale Grid" of X axis and Y axis.

- You can shift the origin of the coordinate system. To proceed, click on the origin and whilst the pointer turns into the shape , hold down the left button of the mouse and move gently the origin.

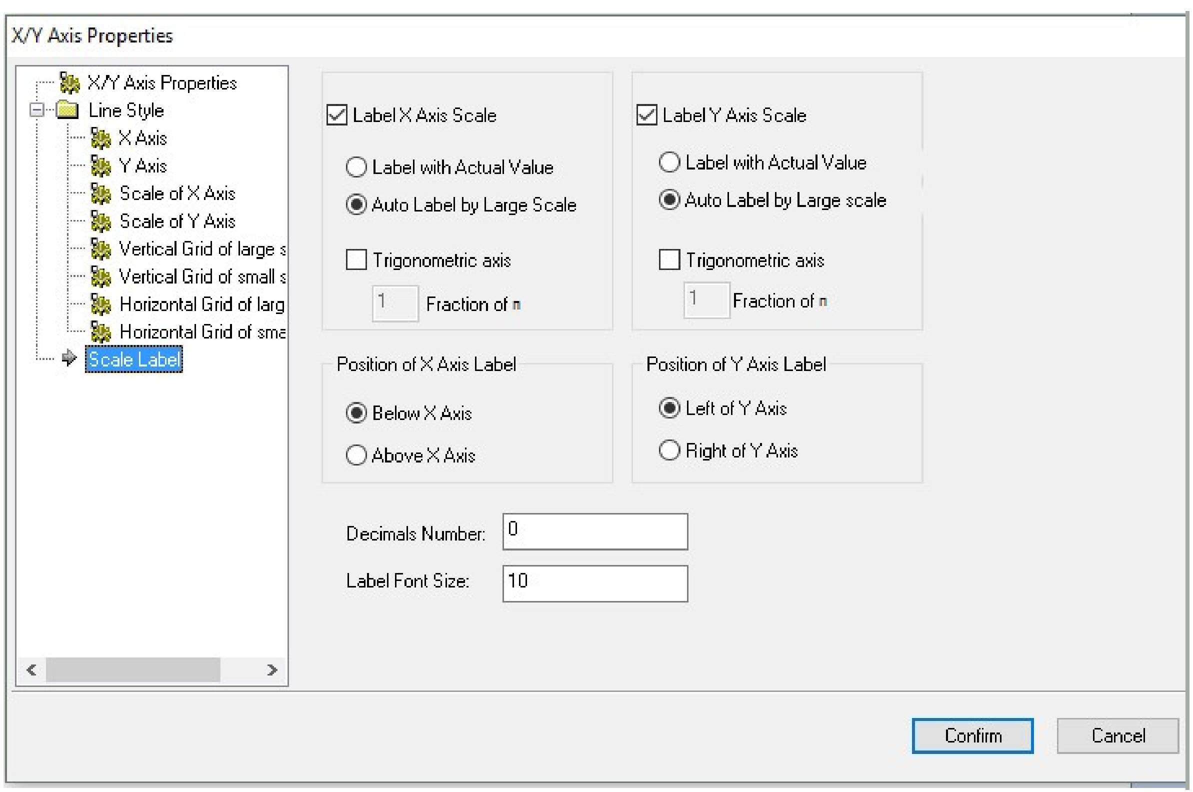
- When the coordinates system is selected, you can hold on any one of the eight small black squares to stretch or enlarge it without modifying the large scale. But the same action through any one of the four black big squares modifies the large scale.
- Thanks to many options in axes properties, you can get different kinds of configurations of the coordinate system as shown below.



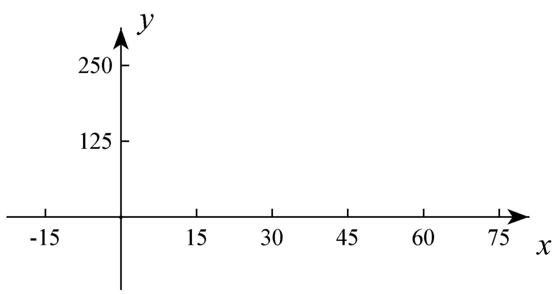




In X/Y axes properties dialog box you can click on "Scale label" to get the following dialog box where you can set up Label axis scale, Label position, the number of decimals of Label and Label font size.



For example, if the large scale increment (in X/Y axis properties dialog box) of X axis is 15 and the large scale increment of Y axis is 125, you may have to check "Label with actual Value" in Label X axis scale and Label Y axis scale options to get Fig1 as axes coordinates. But with the auto label option you will get Fig2.



Figl

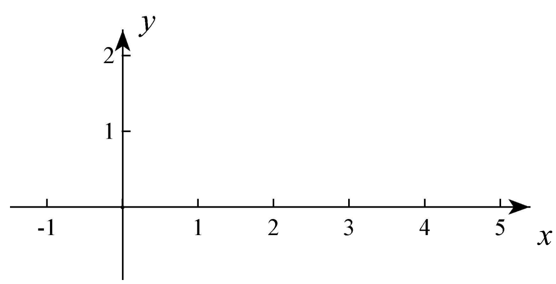


Fig2

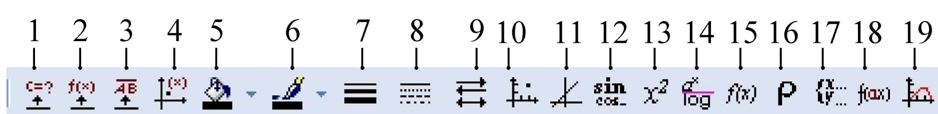
Then for any function graph to be plotted in Fig1, you may have to consider in the function dialog box the following domain: xs = -15 and xe = 75 .

Then for any function graph to be plotted in Fig2, you may have to consider in the function dialog box the following domain: xs = -1 and xe = 5

The "trigonometric axis" option helps to generate points of the related axis such that the distance of two consecutive points is $\frac{\pi}{n}$, where $n \in \mathbb{N}^*$.

2) Corresponding tools of the 2D coordinates system

As soon as a 2D coordinates system is drawn,the tools shown below appear automatically in the geometry toolbar



To find out the function of a tool, just place the pointer on it. After a few seconds, this function is displayed.

1- Define an independent variable; 2- Functional variable; 3- Define a vector; 4- Define a point coordinates; 5- Fill color; 6- Brush color; 7-Line width; 8- Line style; 9- Arrow; 10- Create a coordinates system point; 11- Draw straight line; 12- Graph trigonometric function; 13- Graph conics; 14- Graph exponential and logarithmic functions; 15- Graph function in coordinates system; 16- Polar curve; 17- Graph parametric function; 18- Multi parameters function; 19- Create broken line with given data.

The following table shows the elementary functions list available.

Functions list		
Symbol	Name	Meaning
+ − * /	Addition - Substraction - Multiplication − Division	(3*x−5)/2+1=$\frac{3x-5}{2}$+1
pi	pi	pi=π
^	Power	x^3=x^3
%	Remainder	5%3=2；5.4%2.6=0.2
abs(x)	Absolute value	abs(x)=|x|
sin(x)	Sine	sin(x)=sinx
cos(x)	Cosine	cos(x)=cosx
tan(x)	Tangent	tan(x)=tanx
asin(x)	Sine inverse	asin(x)=arcsinx
acos(x)	Cosine inverse	acos(x)=arcosx
atan(x)	Tangent inverse	atan (x)=arctan$x$ Values domain : [- pi/2, pi/2]
atan2(y, x)	Polar angle (in rads) of P(x,y)	Values domain: [- pi, pi] If y< 0， atan2(y, x)< 0 If y > 0，atan2(y, x) > 0
cosh(x)	Hyperbolic cosine	cosh(x)=cosh$x$
sinh(x)	Hyperbolic sine	sinh(x)=sinh$x$
tanh(x)	Hyperbolic tangent	tanh(x)=tanh$x$
ceil(x)	Ceil function	If n≤x< n+1, ceil(x)=n+1, n ∈ ℤ
deg(x)	Degree function	deg(x) = $\frac{180\,x}{\pi}$
exp(x)	Exponential function	exp(x) = e^x
floor(x)	Floor function	If n≤x< n+1, floor(x)=n, n ∈ ℤ
hypot(x,y)	Hypothenuse function	hypot(x,y) = $\sqrt{x^2+y^2}$
max(x,y)	Max function	max(3,9)= 9
min(x,y)	Min function	min(3,9)=3
Rand(x,y)	Random function	Random values between x and y
mod(x,y)	Modulo function	mod(x,y) = x%y
ln(x)、log(x)	Neperian Logarithm	ln(x)=log(x)=$\log_e$(x)
log10(x)	base 10 logarithm	log10(x)=$\log_{10}$(x)

pow (x,y)	x power y	pow (x. y) = x^y
rad (x)	Radian function	rad (x) = $\frac{x}{180}\pi$
sign (x)	Sign function	If x> 0， sign (x) = 1 If x=0， sign (x) = 0 If x<0， sign (x) = -1
step (x)	Step function	If x ≥ 0， step (x) = 1 If x < 0， step (x) = 0
in (x,r0,r1) or in11 (x,r0,r1)	Interval function	if r0≤x≤r1, in (x,r0,r1) = 1 if x< r0 or x>r1, in (x,r0,r1) = 0
in10 (x,r0,r1)	Left-closed Interval function	if r0<x≤r1, in (x,r0,r1) = 1 if x< r0 or x>r1, in (x,r0,r1) = 0
in01 (x,r0,r1)	Right-closed Interval function	if r0≤x<r1, in (x,r0,r1) = 1 if x< r0 or x≥r1, in (x,r0,r1) = 0
in00 (x,r0,r1)	Open Interval function	if r0<x<r1, in (x,r0,r1) = 1 if x≤r0 or x≥r1, in (x,r0,r1) = 0
sqrt (x)	Square root function	sqrt (x) = $\sqrt{x}$
j0 (x) , j1 (x) , jn (x)	First function of Bessel: degree 0, degree 1, degree n	j0 (x) = J (0, x) j1 (x) = J (1, x) jn (x) =J (n. x)
y0 (x) , y1 (x) , yn (n,x)	Second function of Bessel: degree 0, degree 1, degree n	y0 (x) =Y (0. x) y1 (x) =Y (1, x) yn (x) = Y (n, x)

Notes

- Based on the permitted expression of elementary functions, the corresponding written expressions to $e^x \frac{\sin(x)}{x-1}$ which can appear in the field of "y" are exp (x) *sin (x) / (x-1) , e^x*sin (x) / (x-1) and pow (e,x) *sin (x) / (x-1) .
- The corresponding writing of π is pi. For example, the corresponding expressions of $\frac{\pi x}{2}$ is pi*x/2.
- In trigonometric function or Polar coordinates system dialog box, when the "Angle in pi Radian" box is ticked off (☑ Angle in pi Radian) , then the values of the "*Domain*" are displayed in π radians . To get the configuration of "*Domain*" in numerical values, uncheck the "Angle in pi Radian" box.

- The position of a point or a line in the coordinate system can be modified with the mouse or through their properties.
- The variable under consideration in the graphical representation of a function does not always remain the same. There exist four types of variable: x, y, t and r. For example, an expression such as $y = f(x)$ shows that the accepted variable is x; an expression such as $t = f(r)$ shows that the accepted variable is r; etc.
- Any piecewise function like $F(x) = \begin{cases} F_1(x), \textit{ if } x \in [a_1, b_1] \\ F_2(x), \textit{ if } x \in [a_2, b_2] \\ \dots\dots\dots\dots \\ F_n(x), \textit{ if } x \in [a_n, b_n] \end{cases}$,

  where $[a_i, b_i] \cap [a_j, b_j] = \emptyset$, can be expressed as

  $F(x) = \text{in}(x, a_1, b_1) * F_1(x) + \text{in}(x, a_2, b_2) * F_2(x) + \cdots + \text{in}(x, a_n, b_n) * F_n(x)$
- The function $\text{round}(x, n)$ rounds the value of x to the nearest number, with n significant figures from the decimal point:

$$\text{round}(x, n) = \frac{\text{floor}\left(\left(x + 5 \times 10^{-n-1}\right) \times 10^n\right)}{10^n}$$

- Given a number $x \in \mathbb{R}$ to be truncated and $n \in \mathbb{N}$ the number of elements to be kept behind the decimal point. The truncated value of x defined as $\text{Trunc}(x, \text{n})$ is:

$$\text{Trunc}(x, \text{n}) = \frac{\text{step}(x) * \text{floor}(x * 10^{\text{n}}) + \text{step}(-x) * \text{ceil}(x * 10^{\text{n}})}{10^{\text{n}}}.$$

  - If $x \geq 0$, $\text{Trunc}(x, \text{n}) = \dfrac{\text{floor}(x * 10^{\text{n}})}{10^{\text{n}}}$.
  - If $x < 0$, $\text{Trunc}(x, \text{n}) = \dfrac{\text{ceil}(x * 10^{\text{n}})}{10^{\text{n}}}$.
- The function $\text{Int}(\text{x})$ is represented by $\text{floor}(\text{x})$.
- Let assume that $a_1 \leq a_2$. Then any function like $G(x) = \begin{cases} G_1(x), \text{ if } x \in]-\infty, a_1] \\ G_2(x), \text{ if } x \in [a_2, +\infty[\end{cases}$

  can be represented as: $G(x) = \text{step}(a_1 - x)\, G_1(x) + \text{step}(x - a_2)\, G_2(x)$.

  Also take note of the following expressions:

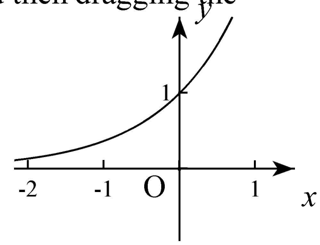
$$H(x) = \text{step}((x - a_1)(x - a_2)) = \begin{cases} 1, \text{ if } x \in]-\infty, a_1] \cup [a_2, +\infty[\\ 0, \text{ if } x \in]a_1, a_2[\end{cases}$$

$$K(x) = \text{step}((-x + a_1)(x - a_2)) = \begin{cases} 0, \text{ if } x \in]-\infty, a_1[\cup]a_2, +\infty[\\ 1 \text{ if } x \in [a_1, a_2] \end{cases}$$

3) Two dimensional common graphical representation

We are going to represent the function: $y = e^x$

Click on the "***Coordinate system***" button in the drawing toolbar, while the pointer turns into a plus " + ", hold down the left button of the mouse and then dragging the mouse, create coordinate system of the desired size. From the tools that appear, click on " Graph Function in Coordinate System". In the dialogue box that opens up, write the appropriate expression of y, that is e^x or exp (x) . Then Click on "OK" to get the curve as shown opposite.



If you want to make some modifications on the curve, then double- click with the mouse on the curve to open its "***Properties***" dialog box where the rotation option helps to rotate the curve about the axes origin.

If you want to make some modifications to the axes of the coordinates, just double click on any axis to open axis X/Y properties dialog box. Then you can carry out the desired modifications.

Note

- Based on the general drawing theory, you can merge two coordinates sytems. To do so, just select one control point of each of the two coordinates sytems and link them.
- You can draw in axes coordinates system many function curves of all types.
- When a line, a circle, an ellipse are geometrically drawn in axes coordinates system, their equations are displayed in object properties dialog box
- You can define variables and use them as parameters in multi parameter function dialog box to create animated function curve.

4) Principles of graphical representation

Let's remind that elementary functions refer to: polynomial, rational, trigonometric, power, exponential, logarithmic and absolute value functions.

a) Functions graphes and types of domain

Generally, the graphical representation of a function $y = f(x)$ is done directly on any domain when its expression does not contain neither the power function $\sqrt[n]{h(x)}$ nor the logarithmic function $\ln(g(x))$.

When $y = f(x)$ contains a power function $\sqrt[n]{h(x)}$, the graphical representation is done on an interval whereas $h(x) \geq 0$.

When $y = f(x)$ contains a logarithmic function $\ln(g(x))$, the graphical representation is done on an interval whereas $g(x) > 0$.

For example, let's consider the following functions:

$$f_1(x) = \frac{x}{x^2 - 1}, \quad f_2(x) = \frac{\sqrt{x}}{x - 1} \quad \text{and} \quad f_3(x) = x - \ln(x - 1).$$

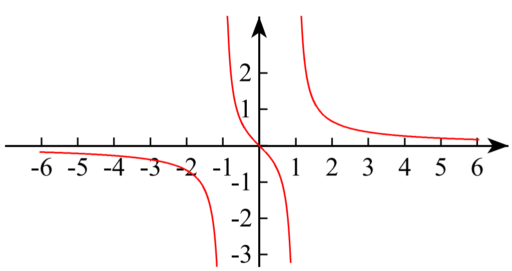
- The graphical representation of f_1 can be carried out directly on $\mathbb{R}$ or on any interval provided that 1 or -1 is not a bound.

- The graphical representation of f_2 can be carried out directly on $[0, +\infty[$ or on any sub-interval provided that 1 is not a bound.

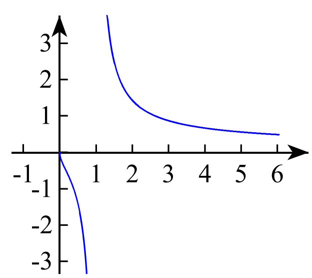
- The graphical representation of f_3 can be carried out directly on $]1, +\infty[$ or on any sub-interval provided that 1 is not a bound.

In a practical way, you just have to represent $f_1(x)$ on $]-6, 6[$, $f_2(x)$ on $[0, 6]$ and $f_3(x)$ on $[1.01, 6]$.

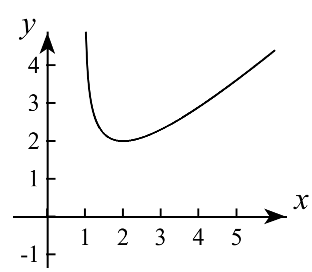
So we have the following results:



Graph of $f_1(x)$



Graph of $f_2(x)$



Graph of $f_3(x)$

b) Note on function power

In ScienceWord and Class as it is the case of most of the scientific software, the power function $x \mapsto \sqrt[n]{x}$ is considered as being defined on $[0, +\infty[$.

But, in reality for all odd values of n, the function power is defined on $\mathbb{R}$. The graphical representation of $y = \sqrt[n]{x}$ in ScienceWord and Class (in case n is odd), use anyone of the three following methods:

i) Given the fact that $\sqrt[n]{x}$ is defined on $[0, +\infty[$ and $\sqrt[n]{-x}$ is defined on $[0, -\infty[$, then we can just consider the expression: $y = \text{sign}(x)\sqrt[n]{\text{sign}(x)\,x} = \text{sign}(x)\sqrt[n]{|x|}$.

ii) We represent $x = g(y) = y^n$ on $\mathbb{R}$.

iii) We represent parametrical function defined as $\begin{cases} x = t^n \\ y = t \end{cases}, \ t \in \mathbb{R}$.

In general, to represent directly the function defined by $y = \sqrt[n]{g(x)}$, where n is an odd integer, it is suitable to just consider its expression $y = \text{sign}(g(x))\sqrt[n]{|g(x)|}$.

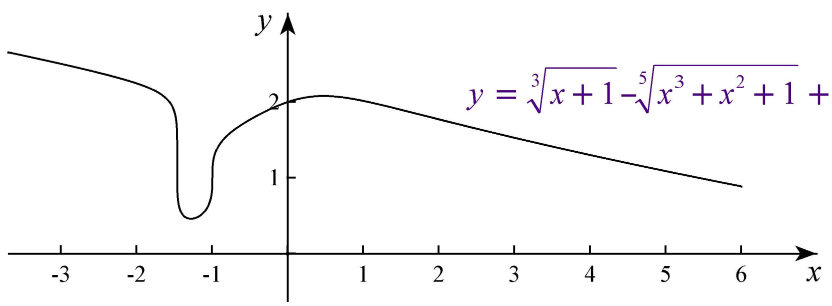
For example,

$y = \sqrt[3]{x+1} - \sqrt[5]{x^3+x^2+1} + 2 = \text{sign}(x+1)\sqrt[3]{|x+1|} - \text{sign}(x^3+x^2+1)\sqrt[5]{|x^3+x^2+1|} + 2.$

The graphical representation uses the following expression:

sign (x+1) * (abs (x+1)) ^ (1/3) -sign (x^3+x^2+1) * (abs (x^3+x^2+1)) ^ (1/5) +2

The graph of the function $y = \sqrt[3]{x+1} - \sqrt[5]{x^3+x^2+1} + 2$ is shown next.



c) Fixing the discontinuous display of an "in" function

The graph of a continuous "in" function whose expression extends over an interval $[a, b]$ appears continuously, if the domain $[c, d]$ on which it is sketched, is included in $[a, b]$. It should be noted that this "in" function is of value 0 outside of $[a, b]$ and could display a discontinuity in a or b if the domain on which it is sketched is not included in $[a, b]$.

Example:

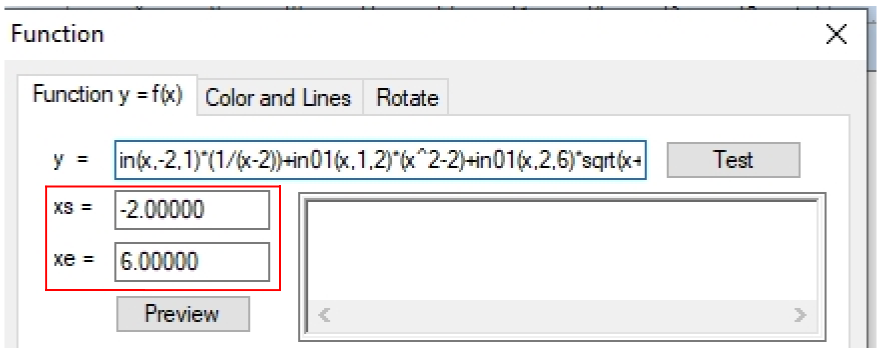
Let consider the function f defined as follows:

$$f(x) = \begin{cases} \dfrac{1}{x-2}, & \text{if } x \in]-\infty, 1] \\ x^2 - 2, & \text{if } x \in [1, 2] \\ \sqrt{x+2}, & \text{if } x \in [2, +\infty[\end{cases}$$

For the purpose of graphical representation over $[-2, 6]$, the corresponding expression of $f(x)$ in "graph function" dialog box, is:

in (x,**-2**,1) * (1/ (x-2)) +in01 (x,1,2) * (x^2-2) +in01 (x,2,**6**) *sqrt (x+2) (§).

Note that the real functions in (x,-2,1) , in01 (x,1,2) , in01 (x,2,6) are not zero function over $[-2, 6]$. The result of the graphical representation over the domain $[-2, 6]$ (as defined in **Fig 1**) , is **Fig 2**.



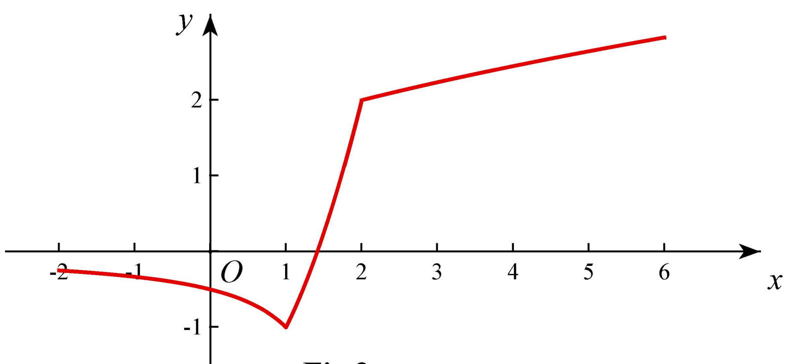


Fig 2

But when the graphe corresponding to expression (§) is sketched over a larger intervalle like $[-2, 7]$, then it should be understood that the functions in (x,-2,1) , in01 (x,1,2) and in01 (x,2,6) are zero function over $]6, 7]$.

In other words, the result of the graphic representation on the domain $[-2, 7]$ (as defined in **Fig 3**) , is **Fig 4**.

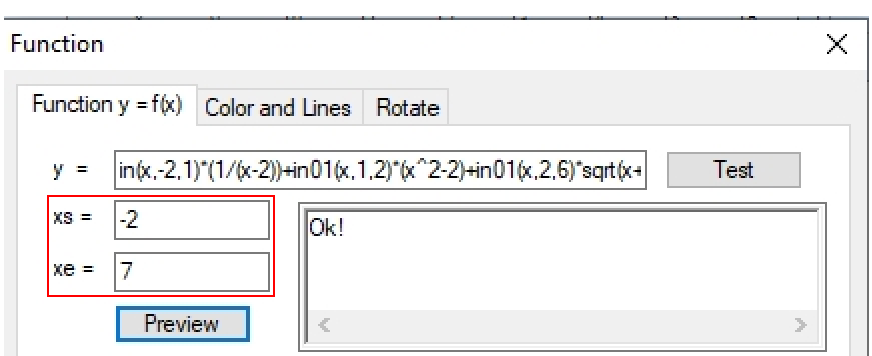


Fig 3

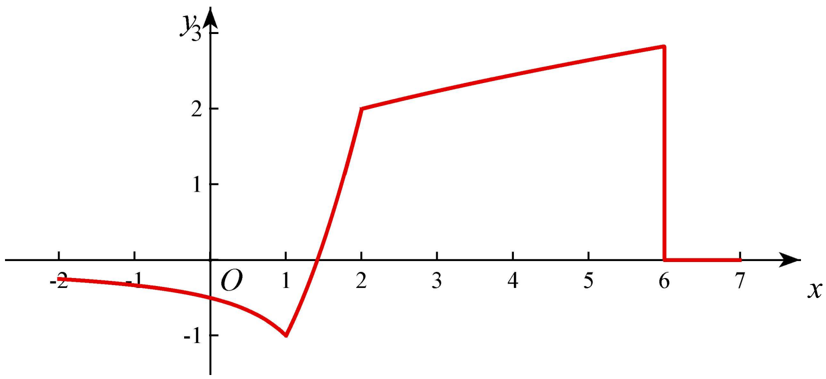


Fig 4

Note: The "step" function graph follows the same logic of continuous display

d) Note on conics equations in 2D coordinates system

In ScienceWord and Class, it is more convenient to use the tool "x^2 Graph second order function" to draw simplified forms of conic defined by $\frac{(x-h)^2}{U}+\frac{(y-k)^2}{V}=1$ (1) or $(x-h)^2=2p(y-k)$ (2) or $(y-k)^2=2p(x-h)$ (3).

However you can double-click on the obtained curve to access the properties dialog box where its angle rotation θ about the axes origin can be set. Let's explain the meaning of such a rotation.

In fact, the equation of the conic should be considered as follow

$$\begin{cases} G(x\cos\theta_0+y\sin\theta_0\,,\,-x\sin\theta_0+y\cos\theta_0)=0 \\ \theta=\theta_0 \end{cases},$$

where θ_0 is the angle rotation value and $G(x,\ y)=Ax^2+Cy^2+Dx+Ey+F=0$ (δ), one of the equations (1), (2) and (3) defined above.

Then the cartesian equation of the conic is given as follow:

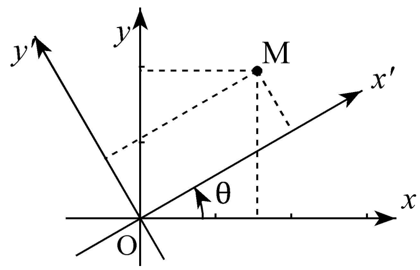
$$A(x\cos\theta_0+y\sin\theta_0)^2+C(-x\sin\theta_0+y\cos\theta_0)^2+D(x\cos\theta_0+y\sin\theta_0)$$
$$+E(-x\sin\theta_0+y\cos\theta_0)+F=0\,.$$

Note that when $\theta_0=0$ (the default value of θ), then this equation is just (δ)

e) Plot conics defined by: $ax^2+bxy+cy^2+dx+ey+f=0$

The general method consists of rotating the axes about an angle value θ around the origin. Then, if M is a point of the conic having $(x,\ y)$ as coordinates before the axes rotation, then the coordinates $(x',\ y')$ of the point M after the axes rotation are found as follow:

$$\begin{cases} x = x'\cos(\theta) - y'\sin(\theta) \\ y = x'\sin(\theta) + y'\cos(\theta) \end{cases}$$



Then the equation of the conic in the new coordinates system is

$Ax'^2 + Bx'y' + Cy'^2 + Dx' + Ey' + F = 0$ where

$A = a\cos^2(\theta) + \frac{b}{2}\sin(2\theta) + c\sin^2(\theta)$;

$B = -(a-c)\sin(2\theta) + b\cos(2\theta)$

$C = a\sin^2(\theta) - \frac{b}{2}\sin(2\theta) + c\cos^2(\theta)$; $D = d\cos(\theta) + e\sin(\theta)$

$E = -d\sin(\theta) + e\cos(\theta)$; $F = f$

We consider the axes rotation where B=0. Then we just have to chose the smallest value of θ such that $\text{cotg}(2\theta) = \frac{a-c}{b}$. Then the new equation is:

$Ax'^2 + Cy'^2 + Dx' + Ey' + F = 0$ (2)

If $a \neq c$, $\tan(2\theta) = \frac{b}{a-c}$ that is $\theta = \frac{1}{2}\text{atan}\left(\frac{b}{a-c}\right)$.

If $a = c$, $\theta = \frac{\pi}{4}\text{rd} = 45°$

Finally, you just have to plot the conic defined by $Ax^2 + Cy^2 + Dx + Ey + F = 0$ and rotate it about θ.

In a practical manner you can use a predefined set of variables *a, b, c, d, e, f* to generate automatically the values of *A, C, D, E, F* and θ. Check the file "General equation" from the folder GLIB (GLIB\Math\Conics\General equation)

f) Parametric equations of conics

The parametric functions of the parabola defined by: $y = x^2$, are

$$\begin{cases} x = t \\ y = t^2 \end{cases}$$

The parametric functions of an elllipse defined by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, are

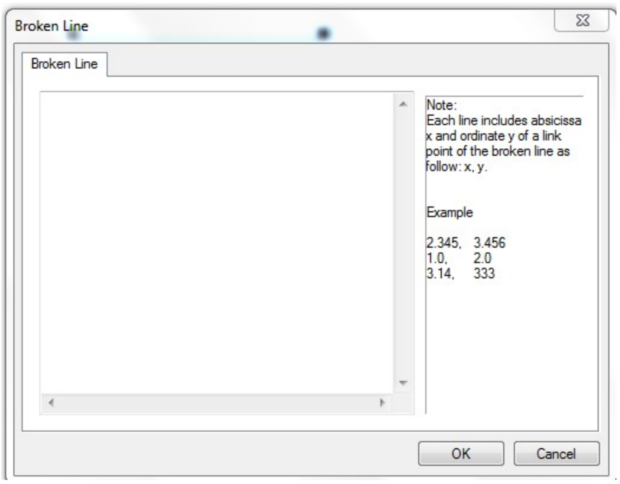
$$\begin{cases} x = a\cos(t) \\ y = a\sin(t) \end{cases}$$

The parametric functions of an hyperbola defined by $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, are

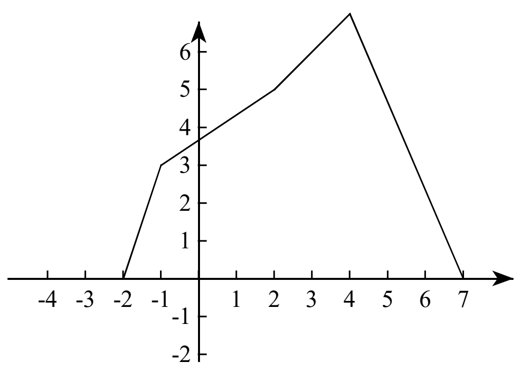
$$\begin{cases} x = \dfrac{a}{\cos(t)} & a > 0, b > 0 \\ y = b\tan(t) \end{cases}$$

5) Joining points in the coordinates system

Click on  tool that appears when the coordinates system is selected. In the dialog box that appears, enter data as shown below.



Then click on "OK" button. The following result is obtained:



Note that you can copy data of the worksheet and paste them directly in the "Broken Line" dialog box and vice versa.

6) Image of a ($\mathcal{C}$) graph by plane isometry

a) Method used

Here the technique used is based on parametrical equations where for any point M (x, y) of ($\mathcal{C}$), we have:

$$\begin{cases} x = h(t) \\ y = g(t) \end{cases} t \in E \quad (E \text{ being a subset of } \mathbb{R}).$$

For example, if $(\mathcal{C})$ is the graph of real function $y=f(x)$, with $x \in D_f$, then we consider the parametrical equations:

$$\begin{cases} x = t \\ y = f(t) \end{cases}, \ t \in D_f \quad (1)$$

- If $(\mathcal{C})$ is the graph of real function $x=f(y)$, with $y \in D_f$, then we consider the parametrical equations:

$$\begin{cases} x = f(t) \\ y = t \end{cases}, \ t \in D_f \quad (1')$$

- If $(\mathcal{C})$ is the graph of polar equation $r=f(t)$, with $t \in D$, then we consider the parametrical equations:

$$\begin{cases} x = f(t)\cos t \\ y = f(t)\sin t \end{cases}, \ t \in D \quad (2)$$

If $(\mathcal{C})$ is the graph of polar equation $t=f(r)$, with $t \in D$, then we consider the parametrical equations:

$$\begin{cases} x = t\cos(f(t)) \\ y = t\sin(f(t)) \end{cases}, \ t \in D \quad (2')$$

b) Analytical expression of plane isometrics

Let consider a plane point $M(x, y)$ and an isometry I and let suppose that $M'(x', y')$ is the point of plane such as $I(M) = M'$.

- If I is the translation with respect to the vector $\vec{u}\begin{pmatrix} a \\ b \end{pmatrix}$, then $\begin{cases} x' = x + a \\ y' = y + b \end{cases}$.

- If I is the central symmetry about the centre $\Omega(x_0, y_0)$, then

$$\begin{cases} x' = -x + 2x_0 \\ y' = -y + 2y_0 \end{cases}.$$

- If I is the symmetry with respect to the axis $y = ax + b$, then

$$\begin{cases} x' = \dfrac{2a}{a^2+1}y - \dfrac{a^2-1}{a^2+1}x - \dfrac{2ab}{a^2+1} \\ y' = \dfrac{a^2-1}{a^2+1}y + \dfrac{2a}{a^2+1}x + \dfrac{2b}{a^2+1} \end{cases}.$$

- If I is the symmetry with respect to the axis $x = b$, then $\begin{cases} x' = 2b - x \\ y' = y \end{cases}$.

- If I is the rotation about the centre $\Omega(x_0, y_0)$ and angle θ, then

$$\begin{cases} x' = (x - x_0)\cos\theta - (y - y_0)\sin\theta + x_0 \\ y' = (x - x_0)\sin\theta + (y - y_0)\cos\theta + y_0 \end{cases}$$

Note: *you can rotate any graph about the axes origin directly through "Rotate" option of this graph properties.*

c) Image of the graph ($\mathcal{C}$)

Let denote $\mathcal{C}' = I(\mathcal{C})$ where I is an isometry .

Assume that the parametrical equations of $\mathcal{C}'$ *are*: $\begin{cases} x = h(t) \\ y = g(t) \end{cases}$.

When replacing in isometry I analytical expression x by $h(t)$ and y by $g(t)$, you would obtain an expression like: $\begin{cases} x' = H(t) \\ y' = G(t) \end{cases}$.

Then , it follows that parametrical equations of $\mathcal{C}'$ are $\begin{cases} x = H(t) \\ y = G(t) \end{cases}$.

<u>Application Example</u>

Let $\mathcal{C}$ be the graph of function $y = f(x)$, which parametrical equations are: $\begin{cases} x = t \\ y = f(t) \end{cases}$.

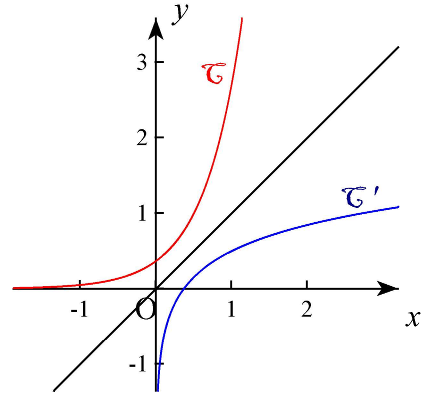
Assume that I is the symmetry with respect to the axis $y = x$, and having parametrical equations $\begin{cases} x' = y \\ y' = x \end{cases}$

When replacing x by t and y by $f(t)$, we have: $\begin{cases} x' = f(t) \\ y' = t \end{cases}$.

Thus, the parametrical equations of $I(\mathcal{C})$ are: $\begin{cases} x = f(t) \\ y = t \end{cases} \quad (i)$.

Remark:

You could also consider the cartesian equation of $\mathcal{C}'$ when such an equation is easy to find out. For example, it is easy to get from (i) the cartesian equation of $\mathrm{I}(\mathcal{C})$, that is $x = f(y) \ \ (ii)$.



In ScienceWord, you can complete $\mathcal{C}'$ through (i) or (ii).

Assume that $\mathcal{C}$ is the graph of function $y = e^{2x-1}$. Then the parametrical equations of $\mathcal{C}' = \mathrm{I}(\mathcal{C})$ are given as follows:

$$\begin{cases} x = e^{2t-1} \\ y = t \end{cases} \quad t \in \mathbb{R}.$$

It is easy to notice that the cartesian equation of $\mathcal{C}'$ is $x = e^{2y-1}$.

The curves $(\mathcal{C})$ and $\mathcal{C}' = I(\mathcal{C})$ are completed opposite.

7) Drawings and coordinates system

When the coordinates system is selected, any geometrical object (line, rectangle, free curve or Bezier curve, etc.) or any experimental tool of chemistry, physics, optics, mechanics, electromagnetism drawn in the active zone of the selection of the coordinates system, is automatically captured. It thus becomes an element of the coordinates system! When these objects are not drawn in the active zone of the coordinates system, you can merge these objects with the coordinate using the " Combine" tool.

Furthermore you can determine the coordinates of any point of this object in the coordinates system! So many practical applications! Without going to the point of drawing an exhaustive list, let just mention that it is henceforth possible to measure rapidly the level of a liquid in a "U Tube", to obtain the coordinates of any point in a geometrical transformation, the impact of a bullet in a shooting experience, an accurate estimate of laboratory experiment results, interesting approximate of solutions of several algebra equations, etc.

8) Filling and functions curves intersection

When function curves and geometrical objects are selected, the " Select and Fill

Region" tool that appears in the Geometry Toolbar task zone helps to fill their intersection or their difference or their union region.

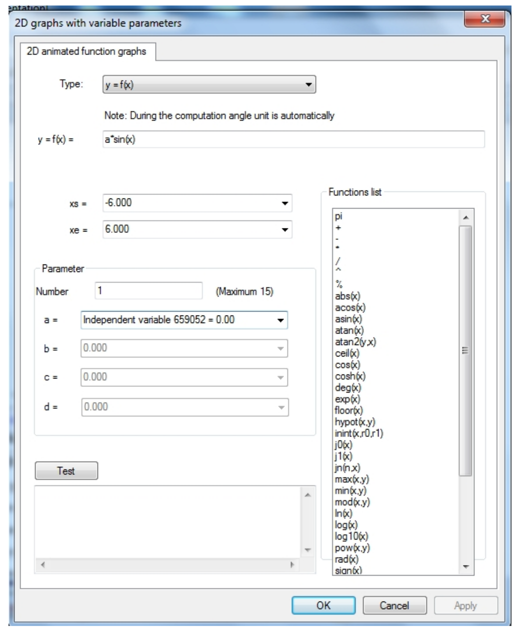
When two function curves or a curve and a line are selected the " " Intersection of two curves" tool that appears in the Geometry Toolbar task zone helps to draw the intersection points.

9) Graph animation

Animation of the graph defined by $y = m\sin(x)$ where $m \in [-2, 3]$.

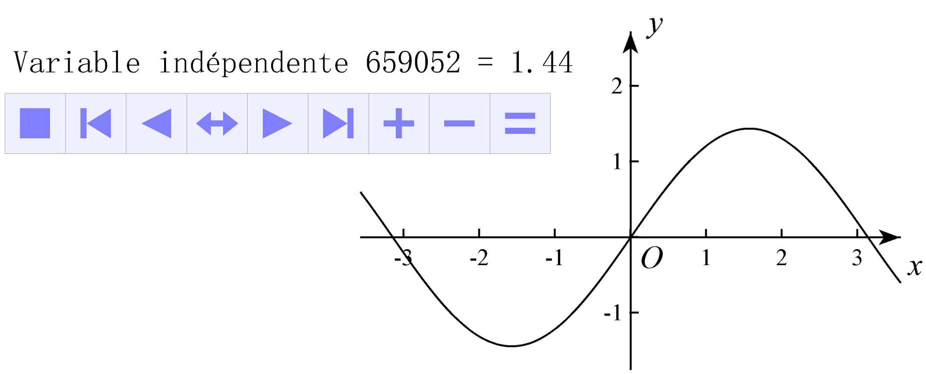
First draw the 2D coordinates system; then define an independent variable from -2 to 3.

Then click on the multiparameter function button ; the dialog box below opens up.



Select the type of function y=f (x) and type a^sin (x) . Then type 1 in the number of parameter box and select a as the independent variable

Click OK button to get the result.

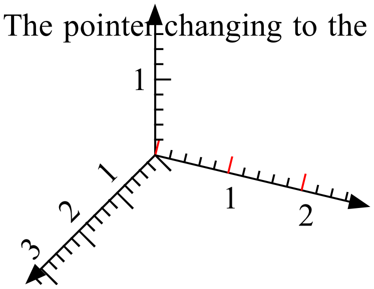


Remark

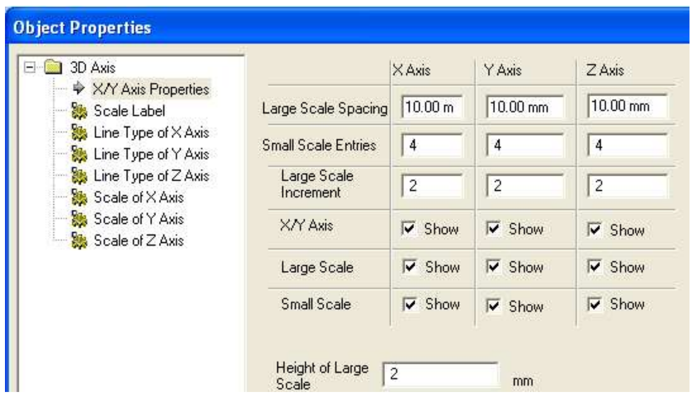
- The boundaries of the function domain can be set as variable values.
- A graph of a function $g(x)$ can also be generated by a variable point M (t, $g(t)$) .where t is a variable parameter. The use of this kind of the variable point proves to be useful when plotting the image by plane transformation of a function graph .

III) Three Dimension graphical representation

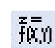
Click on **"3D Coordinates  "** in the Geometry Toolbar. The pointer changing to the shape of a plus " + " on the worksheet, click and hold the left button of the mouse, then drag it slightly to draw the coordinates system as shown the illustration opposite:

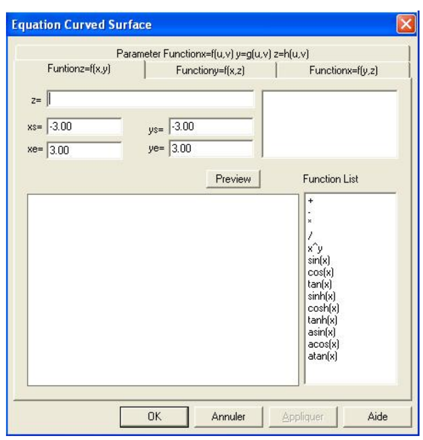


. You can access the following "Object Properties" dialog box of the coordinates system by clicking on "Properties" from the contextual menu.



The above dialog box helps to carry out modifications to the coordinates system.

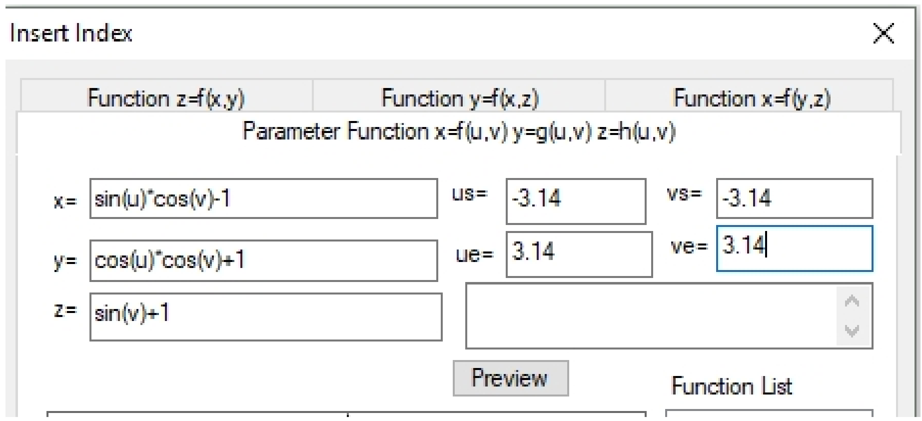
While the coordinates system is active (that is selected) , click on "  Create 3D graphics" tool in the Geometry Toolbar. The following dialog box appears:



Taking into account the list of accepted elementary functions and the definition domain, you can as it is the case in two dimensional coordinates system, plot graphs in three dimensional coordinates system.

As the case in 2 D, the coordinates system 3 D is very flexible. You can in the same way displace the origin of the coordinates system by the mouse.

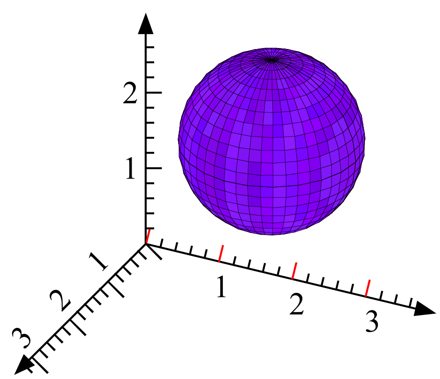
For example, to represent the sphere with radius $r = 1$ and centre $(-1, 1, 1)$, click on the "Parameter Function.." button, and then set the dialogue box as follows.



x = sin (u) *cos (v) -1, y = cos (u) *cos (v) +1, z = sin (v) +1.

u_s= -3.14 , u_e= 3.14, v_s= -3.14, v_e= 3.14.

Click on "OK" button to get the sphere as shown next.



It is possible to sketch in the same coordinates system many surfaces as shown next

A half of sphere defined as

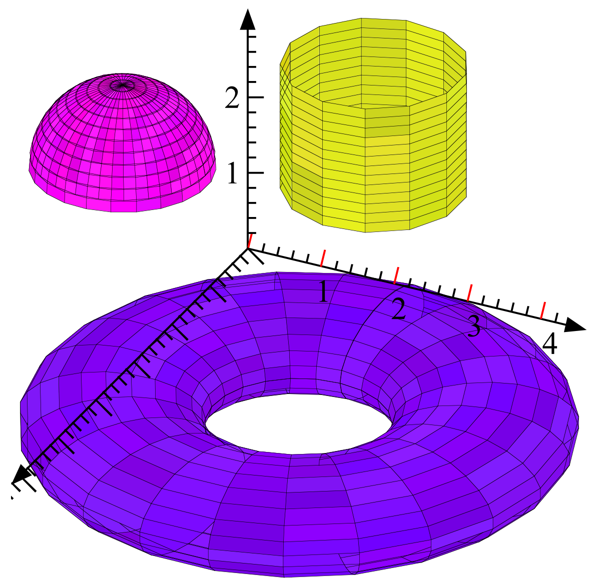
$$\begin{cases} x = \sin(u)\cos(v) - 1 \\ y = \cos(u)\cos(\mathrm{v}) - 2 \quad -3.14 \le u \le 3.14\,;\, 0 \le v \le 3 \\ z = \sin(v) \end{cases}$$

A cylinder defined as

$$\begin{cases} x = \sin(u) + 3 \\ y = \cos(u) + 3 \qquad -3.14 \le u \le 3.14\,;\; 0 \le v \le 1.5 \\ z = v + 3 \end{cases}$$

A torus defined as

$$\begin{cases} x = (3+\cos(v))\cos(u) \\ y = (3+\cos(v))\sin(u) + 0.6 & -3.14 \le u,\ v \le 3.14 \\ z = 0.5\sin(v) - 2 \end{cases}$$



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