ScienceWord and Class dynamic constructions

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Geometrical constructions and animation

1) Introduction

ScienceWord and Class make it easier to build animations thanks to practical tools available in these software. These are the environments where students can apply directly the knowledge they acquired in school through tested activities.

To make it easier for the user to study the content of this book, I would recommend to read first the two books on "Drawing basic notions" and "Drawing and basic dynamic constructions".

In classroom

Teachers can use animation in analytic geometry to give students a tangible, visual way to explore and understand core concepts in mathematics, science and many other fields.

The animation includes various construction methods as described in classroom, the assesment of properties and theorems stated in geometry, the investigation of problems solutions, the exploration of new concepts for the purpose of discovery.

In business, engineering and research

The animation can be used for advert in business or to highlight a project or constructions models with variable parameters in term of engineering design or to approch solutions in term of research activities.

2) Elements of dynamic constructions

a) Variables data

i) Measures

In the following table we are describing the different tools available for measures.

Measures	
Icons	**Specific tasks**
Length	Used to display a length of a segment
Distance	Used to display the distance between two points or a distance from a point to a line
Ratio	Used to find the abscissa of an axis point or the ratio of two segments lengths
Abscissa	Used to plot point on real axis
Perimeter	Used to display the perimeter of a polygon, a circle, an ellipse

Length of arc	Used to display the length of an arc passing three points or an arc defined by two points of a circle or a circle arc ,
Arc measure	Used to display the measure of an arc defined by two points of a circle (or circle arc) or the polar angle of a point of a circle (or circle arc)
Radius	Used to display the radius of a circle
Angle	Used to display the angle measure defined by three points or two rays
Area	Used to display the area of a triangle or a quadrilateral or domain defined by three or four points or any regular polygon.

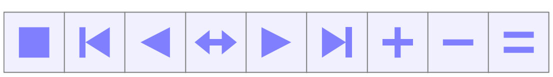
ii) Independent variable

An independent variable is just a data taking a real value that varies in a fixed domain. It could mean anything (a length, an area, a force unit, any kind of measure) . For example, in the resistance formula $R = \rho\ L$, the independent variable could mean anyone of the three variables R, ρ or L .

The independent variable iconl is available in geometry toolbar when an object of the plane is selected. When you click on that icon, the following dialog box appears

The independent variable is displayed as default as follows

Independent variable 25 = 0.00

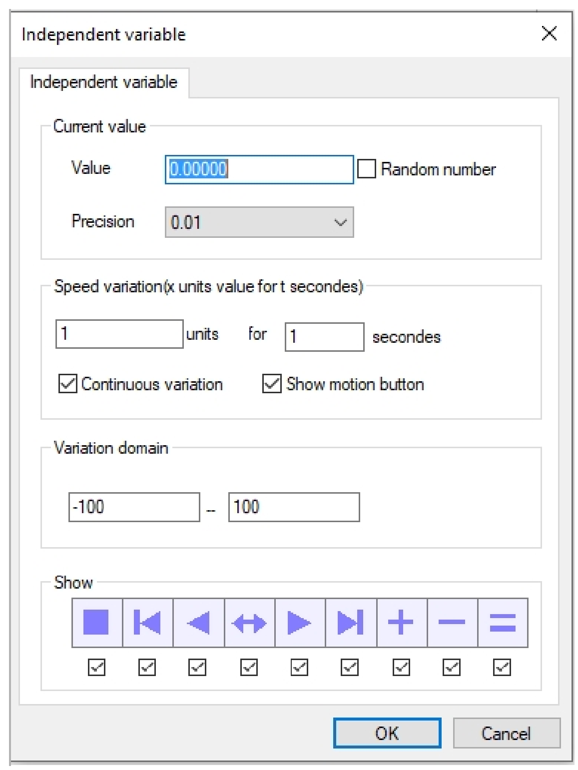


(the number 25 of the variable is not fixed) . When the option "Show motion button" is unchecked, the independent variable is displayed as follows

Independent variable 25 = 0.00

When the option "Random is checked", the independent variable is displayed as follows

Random variable 25 = 0.00



Note: The random variable is animated with the help of its animation button.

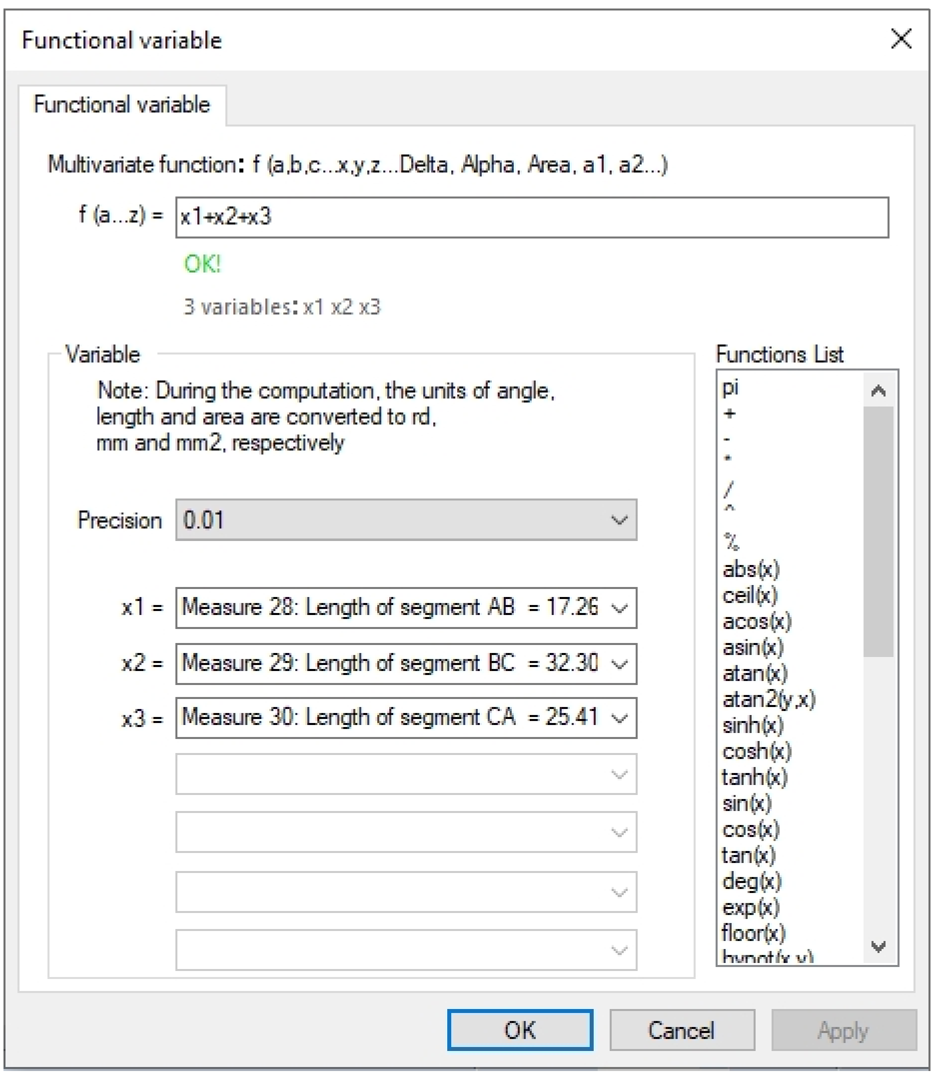
iii) Functional variables

A functional variable is a function of multinumerical data (constant data or variable data) . Let note that a variable data could be an independent variable, a variable distance of two points of a polygon, a variable area of an ellipse, etc.

During the computation of expressions including data like length, area, angle, the conversion is automatically done in mm, mm^2 and radians as noted in the dialogue box. Check more explanation at the end of the document..

The expression of a functional variable f(a, ⋯, z) is made of any kind of variables of types a, b, ⋯, x, y, z, a_1, a_2, ab, velocity, length, etc.

The functional variable tool  is available in geometry toolbar when an object is selected; its dialog box is shown below and the meaning of each element of the function list of that dialog box can be found in the graphical representation book.



Example

$F_1 = xy - y^2 - zx + z^3 - 1$ is a function of 3 variables.

The functions $F_2 = \text{in}(x, r0, r1) = \begin{cases} 1 \text{ if } x \in [r0, r1] \\ 0 \text{ if } \notin [r0, r1] \end{cases}$ and $F_3(x) = \text{step}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$

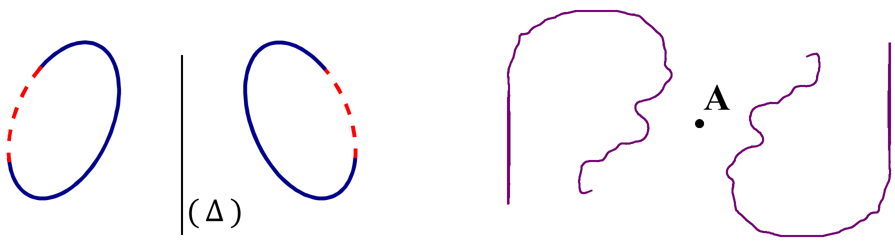
are interval functions of one variable.

b) Geometry transfornation

Just let cite the basic ones: rotation, symmetric, translation, enlargement.

i) Symmetry of an object across a point or across a line

For the moment you can just get the symmetry of one object at once across a point or across a line. All you need to do is to select first the object and second the point or the line. Then click on the tool that pops up automatically in geometry toolbar. The following shows the symmetry across a line (Δ) of an ellipse and the symmetry across a point A of a free hand drawing.



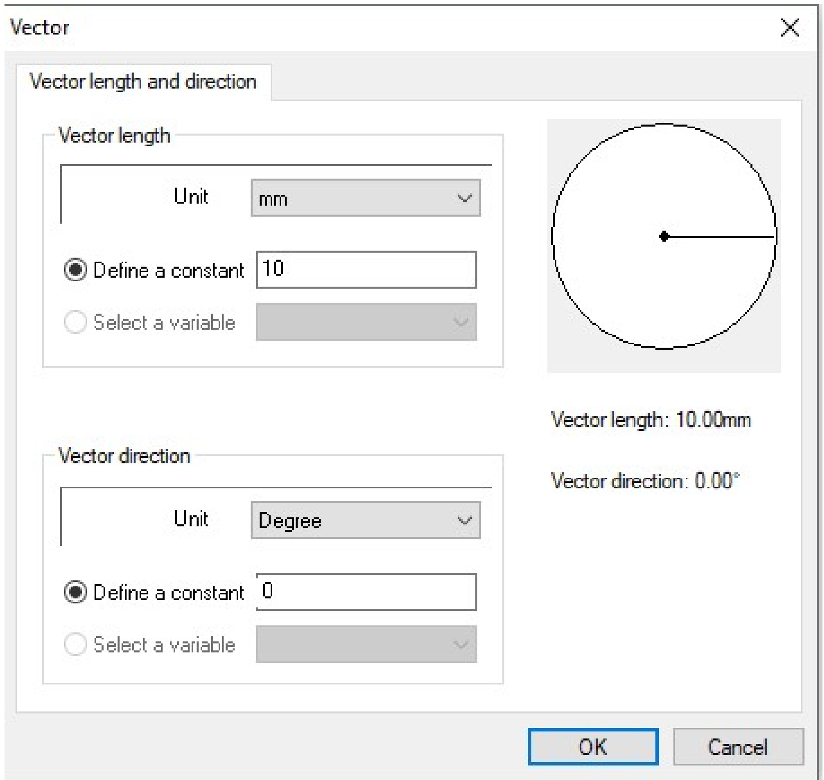
ii) Translation of vector of several objects at once

There is first a need to define a vector:

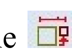
The first method consist just of selecting two points (the order of selection defines the direction) or a line where the direction is from the start point to the end point of that line. Then click on the tool that pops up automatically in geometry toolbar. Then a label is displayed on the screen showing the vector that have been defined.

Next, make sure that the defined vector and the objects to be translated belong to the same drawing region (if not. select them and click on combine tool that pops up automatically in geometry toolbar) . Then click on tranlation vector tool. that pops up automatically in the drawing toolbar and select from the list of vectors defined the one under consideration to get the translation done.

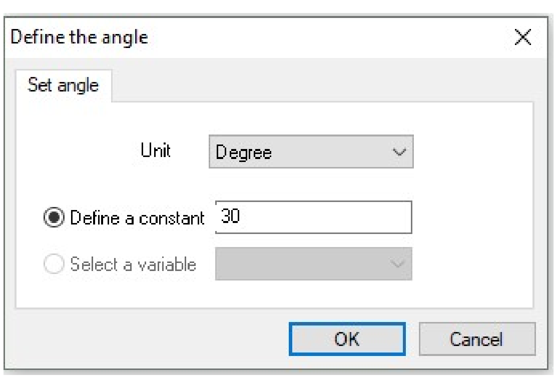
The second method of vector translation definition is based on the use of data (length and angle values) through the " Define a vector length" tool that appears automaticaly in geometry toolbar when an object is selected. Just click on this tool to get the following dialog box for settings.

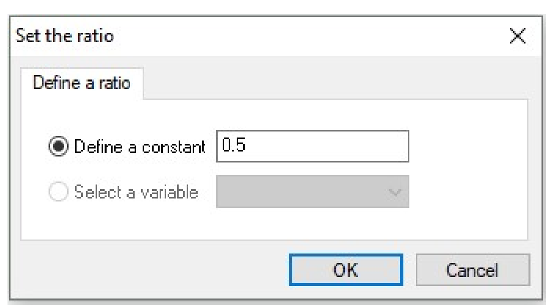


iii) Rotation and enlargement of several objects

Make sure that the objects and the supposed center of the rotation or the enlargement belong to the same drawing region (If not select them and click on  combine tool that pops up automatically in geometry toolbar) . Select them in this order first all the objects and then the supposed center (a point) . Then click on the  rotation tool or the  enlargement tool.

The following shows rotation dialog box and the enlargement dialog box that appear for the appopriate settings





c) Other geometry construction tools

i) Draw an axis point given its abscissa

- When a segment AB is selected (where A is the starting point and B the ending point), the tool helps to plot a point M using its abscissa on the (A, $\overrightarrow{AB}$) axis.
- When two points R and S are in this order selected, the tool helps to plot a point P using its abscissa on the (R, $\overrightarrow{RS}$) axis.

ii) Draw a plane point given its coordinates

The tool " Define a point coordinates" that appears when you draw the 2D coordinates helps to plot a point M using its axes or polar coordinates.

iii) Draw a circle point given its polar angle

The tool " Define a point with a polar angle" appears in geometry toolbar when a circle is selected.. It helps to plot a point on a circle using its polar angle.

d) Control Buttons of animation

There exist four kinds of control animation buttons: Show ∕ hide button, Animation button, Series button and Displace button. You can get them from the sub-menu "Control buttons" of "Insert" menu.

i) Show∕hide button

When objects of a same drawing region are selected, this button is automatically available. It helps to show or hide objects and it proves to be very useful in constructions process and timing.

ii) Animation buttons

When a free object or a displaceable point of an object or an independent variable is selected, the animation button is automatically available. It is particularly very useful for the point animation on a segment, on a polygon, on a circle, on an ellipse or on an arc passing three points. it produces motion at a constant speed.

iii) Displace button

This button is available only when two points of a same drawing region are selected and the second point selected is displaceable. Then you use it to move the first selected point to the level of the second one. it produces motion at a constant speed.

iv) Series button

This button is available when two or more buttons of animation (Show/Hide, Animation button, Displace button) of a same drawing region are selected. You can customize the series button from its motion properties dialog box for a series of actions to be taken simultaneously or in sequences.

3) Principles of dynamic constructions

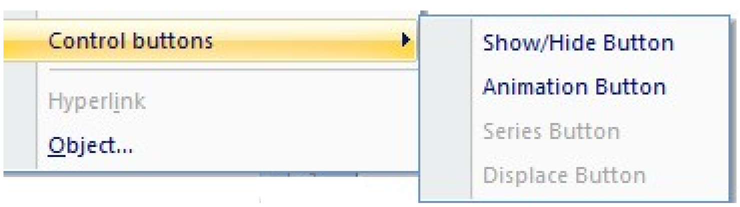
Any animation involving objects in motion starts with a **variable point** of a segment or a polygon or a circle or an ellipse or a **free point** or an **independent variable**.
Note that the middle point of a segment is not a variable point on that segment!

a) Basic examples

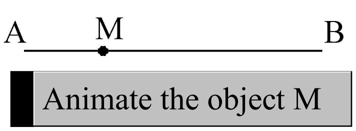
Example 1: Basic animation of a point created through a selection with the mouse

Draw a line-segment AB and click on the tool " Select point of a line " to select a point M (as shown below) .

i. As the point is under selection, click in insert menu on control button and select the animation button.



ii. Click on the animation button to animate the point M

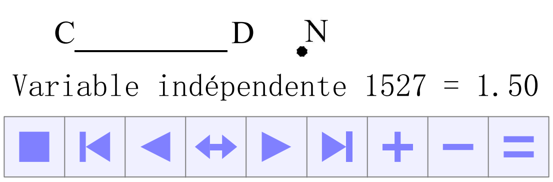


You can double-click on the left black band of the animation button to acces its motion properties for options like "Move to both direction", etc.

Example 2: Basic animation of a point through an independent variable

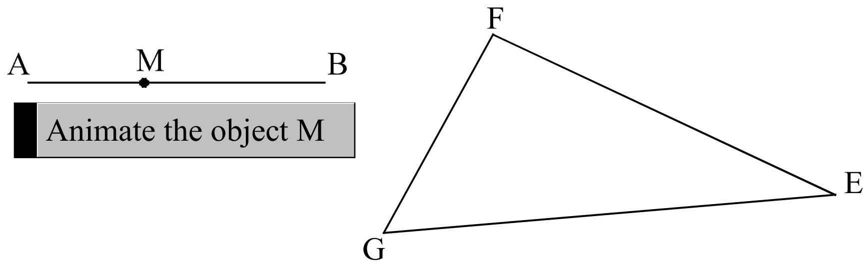
i. Draw a line-segment CD of size 20mm and click on the tool " Define an independent variable "; then set 1.5 as initial value and the domain from -1 to 3.

ii. Select the segment CD and click on the tool " Define an axis point abscissa "; then from the dialog box that pops up, check the option variable and click on the drop-down button to select the independent variable. Then click on OK to get a point N (as shown below) .



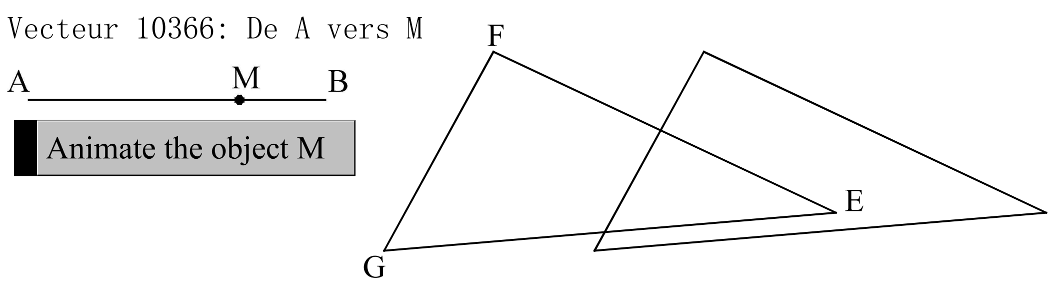
Example 3

1. Let get back the drawing of the example1 and draw a triangle EFG such that EF=5cm, EG=6cm and FG=3cm.



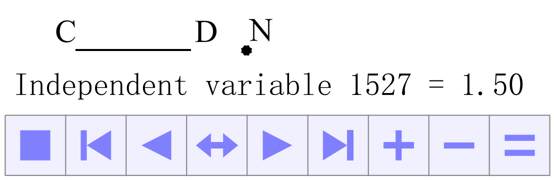
2. Select in this order the end A and the point M; then click in drawing toolbar on the tool "AB Define a vector given two points" to define the vector $\overrightarrow{AM}$.
3. Select the segment AB and the triangle EFG, then click from the drawing toolbar on the tool " Combine " to merge the drawing zone of AB segment and that of EFG triangle.
4. Select solely the triangle EFG and click on the tranlation vector tool to tranlate EFG triangle by vector $\overrightarrow{AM}$
5. Click on the animation button to animate the point M.

The result of this example3 is the figure below



Example 4

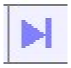
1. Let get back the drawing of the example 2 as shown below

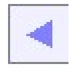


2. Draw the circle centered at C and having the length of CD as radius.
3. Select the circle and click on the tool " Define a point with a polar angle" to draw

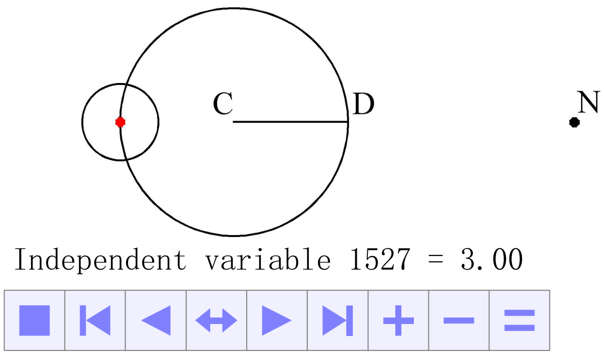
the point Q of the circle correponding to t πrd, where t is the independent variable.

4. Draw the circle of 1cm diameter centered at Q

5. Click on the button  to set the maximum value of the independent variable.

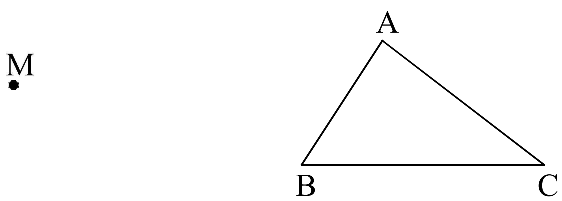
6. Finally click on the decrease button  to animate.

The result of this example 4 is shown below



Example 5: Motion of a free point

1. Click on the icon to draw a free point M; then click on the icon  to draw a triangle ABC as shown below.



2. Select in this order the point M and the point A; then click from insert menu on control button and click on "Displace" button.
3. Select in this order the point M and the point C; then click from insert menu on control button and click on "Displace" button.
4. Animate the movement of the point M to the point A and that of the same point M to the point C.

Remark

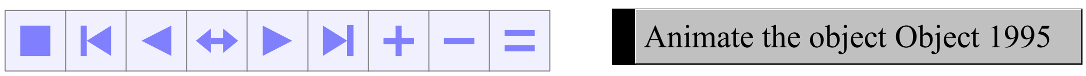
Any end point of a line or any vertex of a polygon that is not bound to any specific condition is also a free point and therefore can be animated with the displace button.

b) Note on objects animation

- Any object including point, polygon, circle, ellipse, etc wich position is not bound by any specific condition is a free object.
- ***You can select a free object, insert its animation button and get it animated radomly.***

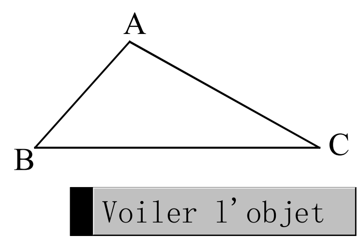
- The following buttons , and help to select a point M on line, polygon, circle or ellipse. The animation button of the point M helps to animate it on the related object.
- You can also select an independent variable and insert its animation button as shown below. This animation helps to animate the value of the independent variable.

Independent variable 1995 = 31.36



c) Other types of basic animation

i) The use of *Show / Hide* button



Example

Draw a triangle ABC and make sure that the triangle is selected. Then click on Show / Hide button from Insert menu on "Control button" . you get a similar result as shown opposite.
You can double-click on the left black band and access the motion properties dialog box for options modification.

ii) The use of *series* button

Example 1

1. Draw a circle of centre O and click on the tool " Define a point with a polar angle" to draw the point A of the circle corresponding to 0°.
2. Select afresh the circle and click on the same tool to draw the point B of the circle correponding to 30°.
3. Repeat the same process to draw the points C of the circle corresponding to 90°
4. Select again the circle and click from geometry toolbar on the tool to select a point P of the circle.
5. Select the point P and insert its animation button; then access the motion properties of the animation button and check the option "one time".

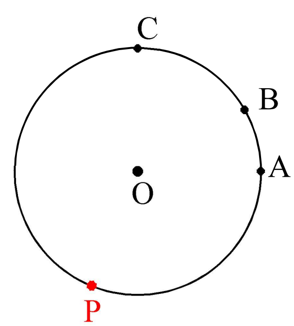


Fig 1

6. Select in this order the point P and the point A and insert the displace button of the point P to the point A.
   Do the same to insert the displace button of point P to point B and the displace button of point P to point C.
7. Select in this order the animation button of P, the displace button of P to A, the displace button of P to B, the displace button of P to C . (Note that to select an animation button, you just need to click on its left black band) . Then click on Series button. from control button in Insert menu

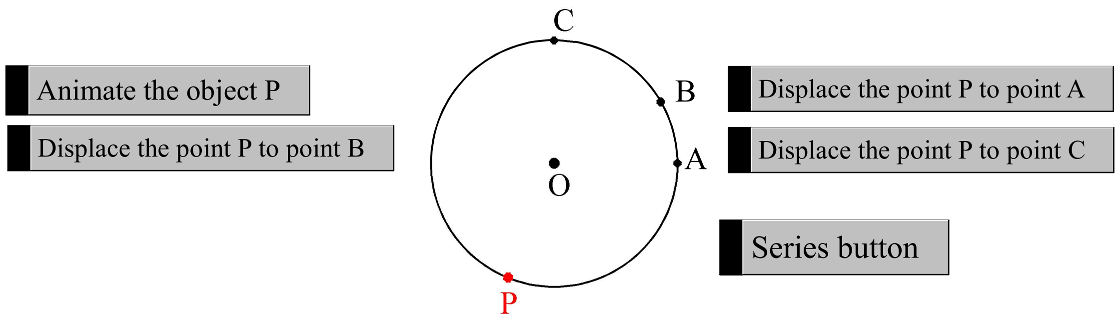
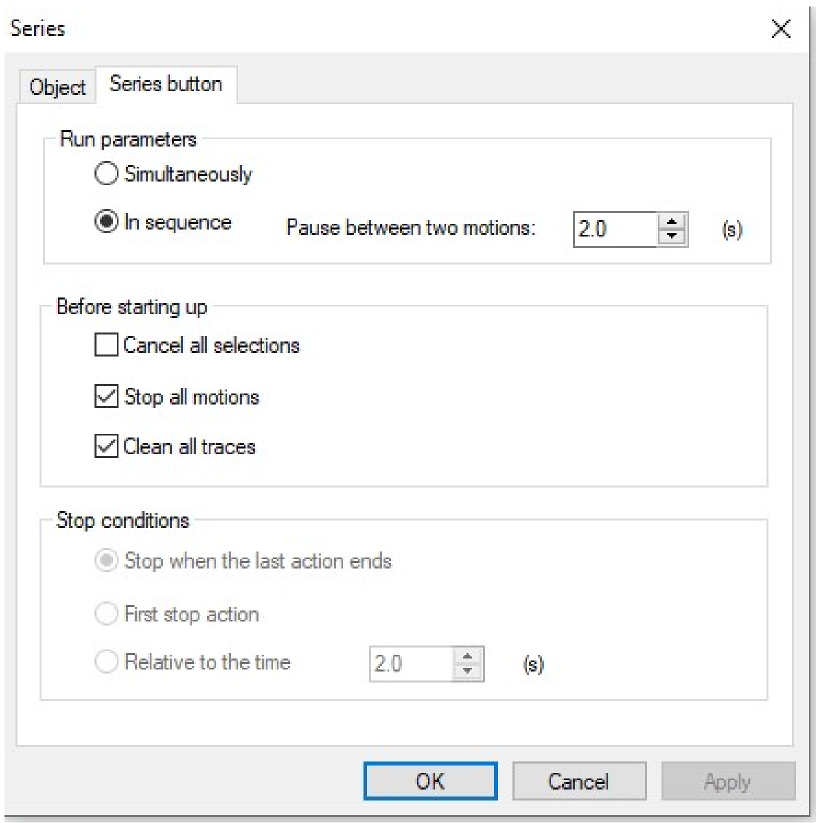


Fig 2

8. Access the motion properties dialog box of the series button and check the option "In sequence" with a pause of 2 secondes as illustrated in the dialog box image below.



Note on animation cancellation

To cancel the above animation produced by the series button, you may need to click several times on undo button from the standard toolbar. The user might find this way of doing thing very clumsy.

A very simple solution consists of drawing an auxiliary point Q of the circle where P should move to as fast as possible.

Then select in this order the point P and the point Q and insert the displace button of point P to point Q. Access the motion properties dialog box and from the speed drop down

button choose the option Ultra fast and change the tag name to cancel (Fig 3) .
Select the point Q and click on the " Hide button" from geometry toolbar. Hide also the animation button of P and all the displace buttons. rename the series button as "Animate the construction". The result is shown below (Fig4) .

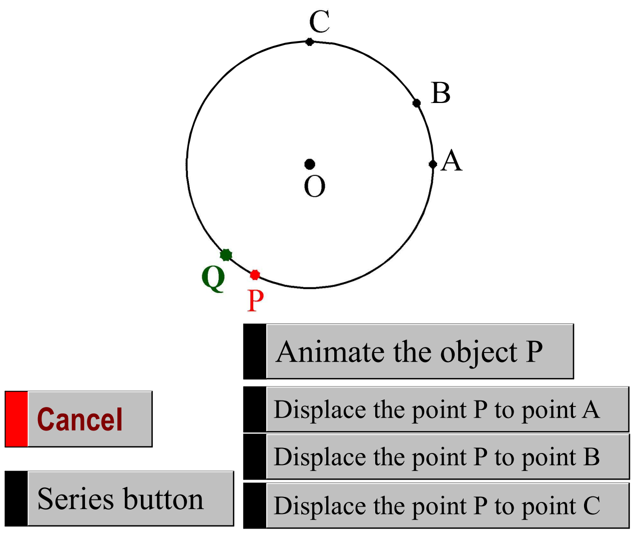


Fig 3

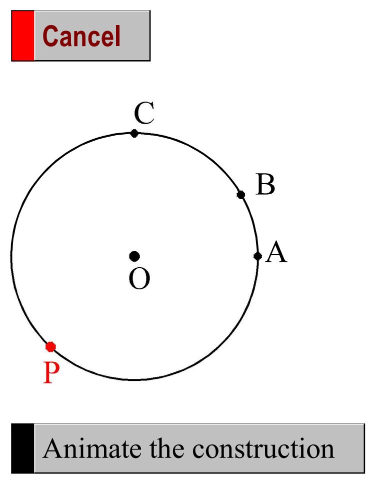
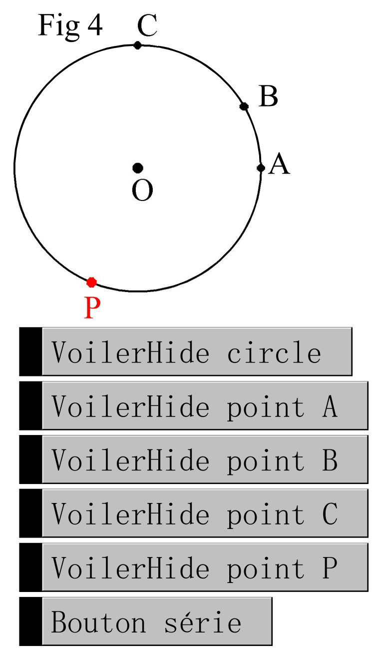


Fig 4



Example 2

In this example we are using the animation to show the steps of construction of Fig1 of the previous example that is the one opposite.

1. Select the circle and insert from the Control buttons in Insert menu its Show / Hide button.
2. Insert the Show / Hide buttons of the points A, B, C, and P..
3. Select in this order the Show / Hide buttons of the circle and the points A, B, C, P and insert the series buttons.
4. Access the motion properties dialog box of the series button and check the option "In sequence" with a pause of 2 seconds
5. Click on the series button to hide the drawing.
6. Click again on the series button to bring back the drawing with all the steps of its construction in proper order.

Note on reversing the order of an animation

To get the reverse order of a movement of a point P through a series button, you need to create a new set of animation buttons of the point P in the reverse order and then create a new series button made of that new set of animation buttons.

Finally a complete dynamic construction made of animations in forth and back could involve many animation buttons. With this kind of classic constructions, you need to hide

the non needed buttons and keep only the ones you will be manipulating.
In fact, ScienceWord and Class indeed offer better new way of animation !

4) The use of functional variable

ScienceWord and Class make available a wide variety of functions including some special ones very helpful in term of animations. Let's just study few examples.

a) The use of the interval function "in (x,r_0,r_1) "

Let recall that: $\mathrm{in}(x,r_0,r_1) = \begin{cases} 1 \; if\, x \in [\, r_0, r_1 \,] \\ 0 \; if\, x \notin [\, r_0, r_1 \,] \end{cases}$, where x is the variable and r0 and r1 are constant real values or variable parameters.

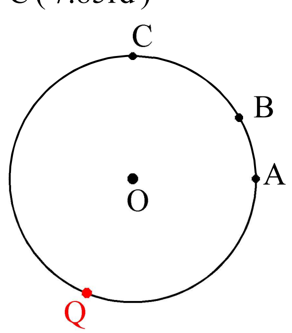
Note that: $\mathrm{in}(x, r_0, r_0) = \begin{cases} 1\,,\; if\, x = r_0 \\ 0,\; if\, x \neq r_0 \end{cases}$ and therefore

$$\mathrm{in}(x,r_0,r_1) - \mathrm{in}(x,r_0,r_0) = \begin{cases} 1,\; if\, x \in \,]\, r_0, r_1 \,] \\ 0,\; if\, x \notin \,]\, r_0, r_1 \,] \end{cases} = \begin{cases} 1,\; if\, x \in \,]\, r_0, r_1 \,] \\ 0,\; if\, x \in \,]-\infty, r_0 \,] \cup \,]\, r_1\,, +\infty\, [\end{cases}$$

Example1

Let consider again a motion anticlockwise of a point P starting its journey from the position of the point Q and first running a full circumference then coming back at its initial position Q, then pausing 2s and now moving to the position of A with another pause of 2s, then moving to the position of B with another pause of 2s and finally moving to its final destination to the position of the point C.

First get the polar angle value in rd of Q, A, B and C.
The journey of P can be described as follows:
Q (-1. 96rd) ⟶ Q (-1. 96rd+6. 28rd = 4. 32rd) ⟶ A (6. 28rd) ⟶ B (6. 80rd) ⟶ C (7.85rd)



Measure of C = 1. 57rad
Measure of B = 0. 52rad
Measure of A = 0. 00rad
Measure of Q = -1. 96rad

Then we have to link a time t to each sequence of motion including the waiting time.and make sure all this come in a total of time T.
Then let define this time as independent variable on a domain **D** from 0 to 30 secondes, where **D** = [0, 12 [∪ [12, 14 [∪ [14, 19 [∪ [19, 21 [∪ [21, 24 [∪ [24, 26 [∪ [26, 30]
In other words, we are considering the following model

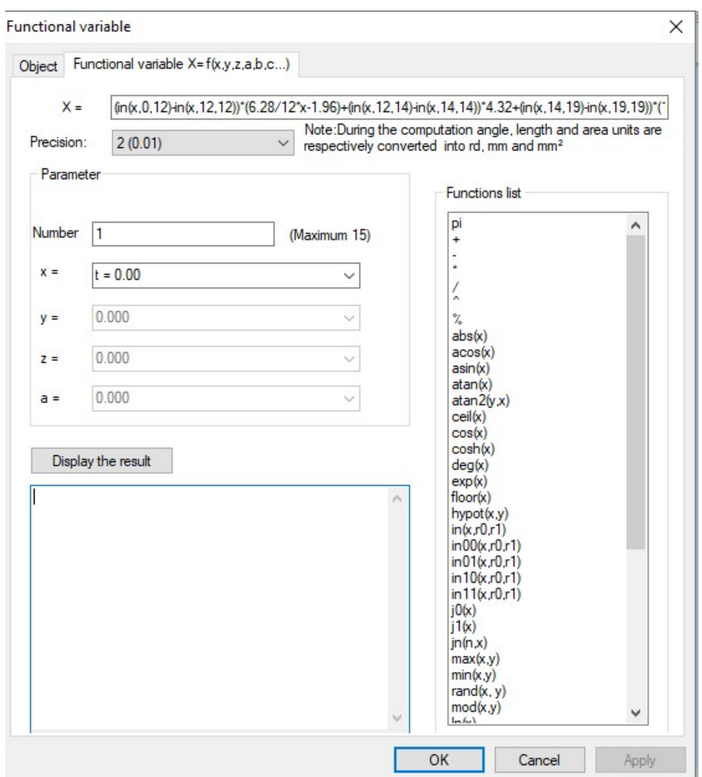
$Q(-1.96rd) \xrightarrow{12\,s} Q(4.32rd) \xrightarrow{5\,s} A(6.28rd) \xrightarrow{3\,s} B(6.80rd) \xrightarrow{4\,s}$ $C(7.85rd)$.

Now we have to consider the type of motion for each sequence, for example a uniform one. This is a king of linear motion defined by $X(t) = \frac{X_2 - X_1}{t_2 - t_1}(t - t_1) + X_1$ where X is the polar angle value of the point P at a time t and X_1 and X_2 the boundaries values of an animation sequence domain at the time t_1 and t_2, that is $X_1 = X(t_1)$ and $X_2 = X(t_2)$.

Then the polar angle value of the point P at a time t is defined as follows:

$$f(t) = (in(t,0,12) - in(t,12,12))\left(\frac{6.28}{12}t - 1.96\right) + (in(t,12,14) - in(t,14,14)) \times 4.32$$
$$+ (in(t,14,19) - in(t,19,19))\left(\frac{1.96}{5}(t-14) + 4.32\right) + (in(t,19,21) - in(t,21,21)) \times 6.28$$
$$+ (in(t,21,24) - in(t,24,24))\left(\frac{0.52}{3}(t-21) + 6.28\right) + (in(t,24,26) - in(t,26,26)) \times 6.80$$
$$+ in(t,26,30) \times \left(\frac{1.05}{4}(t-26) + 6.80\right)$$

Now we have to set this expression as functional variable. Make sure that the drawing is selected and click on the "f(x) functional variable". Then set $X = f(x)$ and $x = t$.



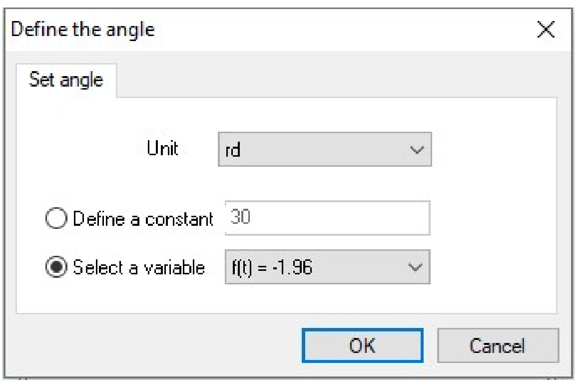
In fact, the expression that is recognized in the functional variable dialog box is:

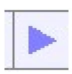
(in (x,0,12) -in (x,12,12)) * (6.28/12*x-1.96)
+ (in (x,12,14) -in (x,14,14)) *4.32
+ (in (x,14,19) -in (x,19,19)) * (1.96/5* (x-14) +4.32)
+ (in (x,19,21) -in (x,21,21)) *6.28
+ (in (x,21,24) -in (x,24,24)) * (0.52/3* (x-21) +6.28)
+ (in (x,24,26) -in (x,26,26)) *6.80
+in (x,26,30) * (1.05/4* (x-26) +6.80)

where the variabke x is t.

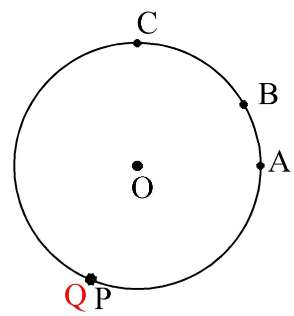
Click Ok to get the variable. Then access to the motion propertie dialog box of the variable and change the tag to f (t) and click OK.

To draw the point P, select the circle and click on the tool " Define a point with a polar angle". Then in the dialog box that opens up, select the unit rd , check the variable option and select from the drop-down button the variable f (t) Then click oK..



Click increase button  to animate the point P.

The advantage of this method is that you have an easy and full control over the animation, where any position or the speed of P can be set, the motion can be done back and forth with a single animation button.



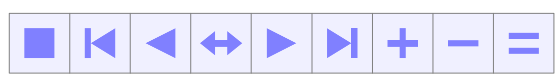
Measure of C = 0.50πrad

Measure of B = 0.17πrad

Measure of A = 0.00πrad

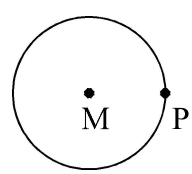
Measure of Q = -0.62πrad f(t) = -1.96

t = 0.00



Example 2: the use of functional variable to hide or show an object (example of a circle)

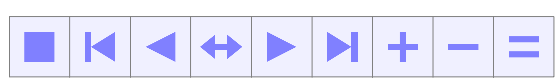
1. Draw a point M and define an independent variable t from 0 to 2. with a precision 0.
2. Define the functional variable V=in (t,1,2) .
3. Define the vector having V cm as length and 0° as direction.
4. Translate the point M by the defined vector to get the point P.
5. Draw the circle centered at M and passing P.

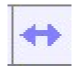


V = 1.00

Vector 3178:Length=1.00cm, direction=0.00°

t = 1



6. Click on the bidirectional button  to show or hide the circle.

Note: the use of independent and functional variables in animation

In a complex animation several functional variables could be defined in relation with an independent variable. You may sometimes also need to define several independent variiables and the "series" animation button.

b) The use of functions in (x,r_0,r_1) , step (x) in animation

The animations can be done easier with a flexible control when the use as variables of the functions in (x, r_0, r_1) , step (x) , sign (x) is well known. We wish to create an awarness of the importance of these functions through some simple examples.

i) The function in (x,r_0,r_1)

Let recall that:

$$\text{in}(x,r0,r1)=\begin{cases}1. \text{ if } x\in[r_0, r_1]\\ 0, \text{ if } x\in]-\infty, r_0[\cup]r_1, +\infty[\end{cases}. \text{ or } 1\text{-in}(x,r0,r1)=\begin{cases}0, \text{ if } x\in[r_0, r_1]\\ 1, \text{ if } x\notin[r_0, r_1]\end{cases}$$

A combined use of in (x,r0,r1) and (1-in (x,r0,r1)) could be helpful as shown below.

$$\text{in}(x,a_1,a_2)*(x\text{^}2\text{-}1)+(1\text{- in}(x,a_1,a_2))*\left(e^x\text{-}\frac{1}{x}\right)=\begin{cases}x^2-1, \text{ if } x\in[a_1, a_2]\\ e^x-\frac{1}{x}, \text{ if } x\in]-\infty, a_1[\cup]a_2, +\infty[\end{cases}$$

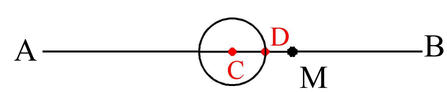
Let's also mention that for any function h (x) and any variable parameters a and b,

$$\text{in}(h(x), \min(a,b), \max(a,b))=\begin{cases}1. \text{ if } h(x)\in[\min(a,b), \max(a,b)]\\ 0, \text{ if } h(x)\in]-\infty, \min(a,b)[\cup]\max(a,b), +\infty[\end{cases}$$

Note that min and max are available in the functions list and therefore can be used.

Example1: animation in a variable domain

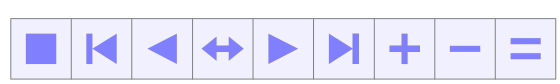
1. Draw a segment AB of 5cm length and select a point M (not the middle) of the segment; then display the abscissa m of M and insert its animation button.
2. Define an independent variable t from -6.28 to 6.28
3. Define the functional variable R=in (x, min (sin (y) ,cos (y)) ,max (sin (y) ,cos (y))) *x^2. where x is the abscissa of the point M and y the independent variable t.
4. Define the vector having R cm and direction 0°.
5. Draw the centre C of the segment AB and translate it by the defined vector to get a point D.
6. Draw the circle centered at C and passing D

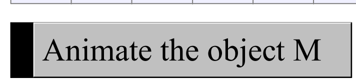


R = 0.43 Abscissa of M = 0.6582

Length=0.00cm, direction=0.00°

t = 5.44





7. Animate solely the point M.
8. Animate solely the independent variable
9. Animate the point M and the independent variable.

Example 2: animation on mutiple curves

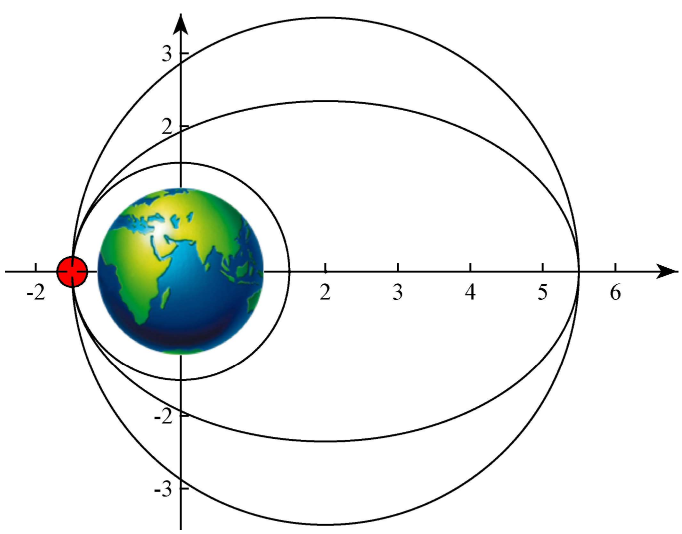
Let note that you can click on the circle icon  to draw a circle or click on the ellipse icon  to draw an ellipse in 2D coordinates and find that circle and ellipse equations from object properties dialogue box.

Then, let consider the following model made of two circles and an ellipse internally tangent.

The equations of the smaller circle, the ellipse and the bigger circle are given as follow:

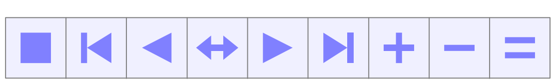
$$\begin{cases} x = 1.5*\cos(t) \\ y = 1.5*\sin(t) \end{cases} (1) \quad \begin{cases} x = 3.5*\cos(t)+2 \\ y = 2.35*\sin(t) \end{cases} (2) \quad \begin{cases} x = 3.5*\cos(t)+2 \\ y = 3.5*\sin(t) \end{cases} (3)$$

In this model, a red point moves from the common tangent point through the smaller circle, run a full circumference and comes back to its initial position, then moves through the ellipse, run a full circumference and comes back to its initial point, then moves through the bigger circle and comes back to its initial point, then moves through the ellipse another full circumference and comes back to its initial position, then moves through the smaller circle and comes back to its initial position.



X = -1.50 Y= -0.00

t = -3.14



The motion of the red point can be described as follow.

The red point runs

- the smaller circle for an angle t ∈ [-3.14 rd, 3.14 rd [∪ [21.98 rd, 28.26 rd]
- the ellipse for an angle t ∈ [3.14 rd, 9.42 rd [∪ [15.70 rd, 21.98 rd [
- the bigger circle for an angle t ∈ [9.42 rd, 15.70 rd [

Then the coordinates of the red point is a variable poin M (X, Y) where

X=1.5* (in10 (x,-3.14,3.14) +in (x,21.98,28.26)) *cos (x) + (in10 (x,3.14,9.42) +in10 (x,15.70,21.98)) * (3.5*cos (x) +2) + (in10 (x,9.42,15.70)) * (3.5*cos (x) +2)

Y=1.5* (in10 (x,-3.14,3.14) +in (x,21.98,28.26)) *sin (x) +2.35* (in10 (x,3.14,9.42) + in10 (x,15.70,21.98)) *sin (x) +3.5*in (x,9.42,15.70) *sin (x)

and x be an independent variable with domain [-3.14, 28.26]

ii) The function Step (x)

Let recall that:

$$\text{Step}(x) = \begin{cases} 1, \text{ if } x \geq 0 \\ 0, \text{ if } x < 0 \end{cases} \quad \text{or } 1-\text{step}(x) = \begin{cases} 0, \text{ if } x \geq 0 \\ 1, \text{ if } x < 0 \end{cases}$$

$$\text{step}(-x) = \begin{cases} 1, \text{ if } x \leq 0 \\ 0, \text{ if } x > 0 \end{cases} \quad \text{or } 1-\text{step}(-x) = \begin{cases} 0, \text{ if } x \leq 0 \\ 1, \text{ if } x > 0 \end{cases}$$

In general if a_1 and a_2 are two reals such that $a_1 \leq a_2$, then

$$\text{Step}(x-a_1) = \begin{cases} 1, \text{ if } x \geq a_1 \\ 0, \text{ if } x < a_1 \end{cases} \quad \text{or } 1-\text{Step}(x-a_1) = \begin{cases} 0, \text{ if } x \geq a_1 \\ 1, \text{ if } x < a_1 \end{cases}$$

$$\text{Step}(-x+a) = \begin{cases} 0, \text{ if} > a_1 \\ 1, \text{ if } \leq a_1 \end{cases} \quad \text{or } 1-\text{Step}(-x+a) = \begin{cases} 1, \text{ if} > a_1 \\ 0, \text{ if } \leq a_1 \end{cases}$$

$$\text{step}((x-a_1)(x-a_2)) = \begin{cases} 1, \textit{ if } x \in]-\infty, a_1] \cup [a_2, +\infty[\\ 0, \textit{ if } x \in]a_1, a_2[\end{cases}.$$

$$\text{Then, } 1-\text{step}((x-a_1)(x-a_2)) = \begin{cases} 0, \textit{ if } x \in]-\infty, a_1] \cup [a_2, +\infty[\\ 1, \textit{ if } x \in]a_1, a_2[\end{cases}$$

$$\text{step}((-x+a_1)(x-a_2)) = \begin{cases} 0, \textit{ if } x \in]-\infty, a_1[\cup]a_2, +\infty[\\ 1 \textit{ if } x \in [a_1, a_2] \end{cases} = \text{in } (x, a_1, a_2).$$

$$\text{Then, } 1-\text{step}((-x+a_1)(x-a_2)) = \begin{cases} 1, \textit{ if } x \in]-\infty, a_1[\cup]a_2, +\infty[\\ 0, \textit{ if } x \in [a_1, a_2] \end{cases} = 1-\text{in } (x, a_1, a_2)$$

iii) Defining a discrete function

Note that: $\operatorname{sign}(x-\operatorname{floor}(x)) = \begin{cases} 0, \text{ if } x \text{ is an integer} \\ 1, \text{ if } x \text{ is not an integer} \end{cases}$ or

$$1-\operatorname{sign}(x-\operatorname{floor}(x)) = \begin{cases} 1, \text{ if } x \text{ is an integer} \\ 0, \text{ if } x \text{ is not an integer} \end{cases}$$

Then we can define on any domain D a discrete function using the "in" function. For example if $D=[2,7]$, the discrete function on D can be written as follow

$$\operatorname{in}(x,2,7) * (1\text{-}\operatorname{sign}(x\text{-}\operatorname{floor}(x))) = \begin{cases} 1, \text{ if } x \text{ is an integer of } [2,7] \\ 0, \text{ if } x \in [2,7] \text{ but } x \text{ is not an integer} \\ 0, \text{ if } x \notin [2,7] \end{cases}$$

c) Meaning of other available functions

The Remainder and Modulo functions

If a and b are two natural numbers, we can always find a unique couple of natural numbers (q, r) such that a=bq+r, while $0 \le r < b$.

r is called the remainder of the division a/b. In ScienceWord and Class the notation is: $a\%b = r$.

- The computations are extended to negative numbers as follow: $a\%(-b) = r$; $-a\%b = -r$ and $-a\%(-b) = -r$.
- If c is another number such that $c\%b = r$, that is a and c have the same remainder in the division a/b, then it is said that a is congruent to c modulo b and noted:$a \equiv c\,(b)$.
- In ScienceWord and class the function modulo is defined as: $\operatorname{mod}(a, b) = a\%b = r$.

Note: In ScienceWord and Class, the choice of a and b is extended to real numbers set $\mathbb{R}$. In this case q is an integer and r is a decimal number.

Example 1: Clock settings

1. Define three independent variables V_H , V_M and V_S with the following settings

V_H:

Value	Speed	Domain
2 Precision: 0.01	1 unit per 3600 s Discontinuous	From 0 to 24

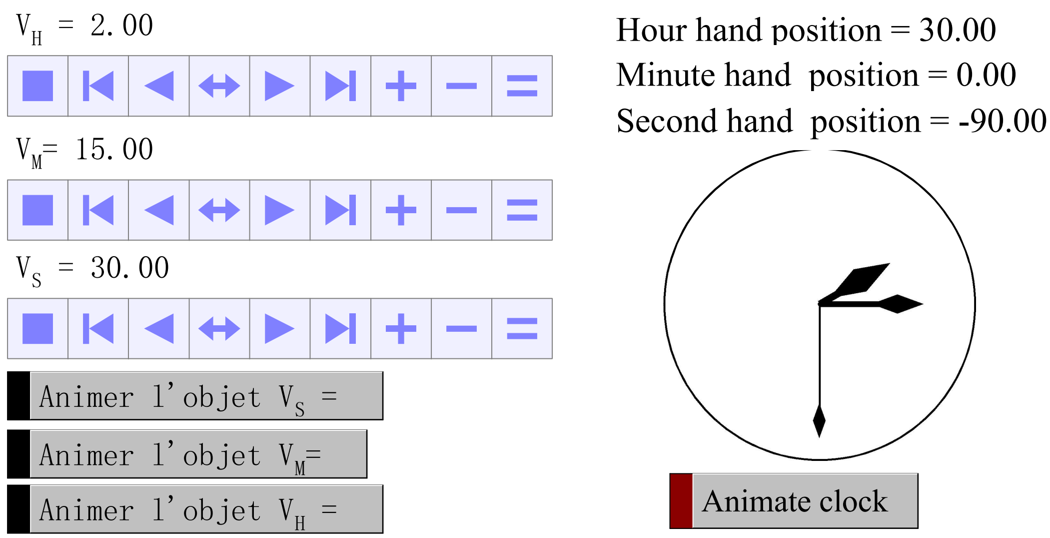
V_M:

Value	Speed	Domain
15 Precision: 0.01	1 unit per 60 s Discontinuous	From 0 to 1440

V_S:

Value	Speed	Domain
30 Precision: 0.01	1 unit per second Discontinuous	From 0 to 86400

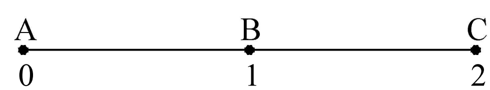
2. Define the following functional variables H= 90 - mod (x, 12) *30 where $x=V_H$; M = 90 - mod (x, 60) *6 where $x=V_M$ and S= 90 - mod (x, 60) *6 where $x=V_S$.
3. Select the circle and click on the tool " define ··· with the ploar angle" to set the position of the point corresponding to H°. Repeat the same process to set the position of the points corresponding to M° and S°.
4. Draw the points X, Y, Z such that $\overrightarrow{OX} = 0.3\ \overrightarrow{OH}$, $\overrightarrow{OY} = 0.5\ \overrightarrow{OM}$, $\overrightarrow{OZ} = 0.75\ \overrightarrow{OS}$
5. Insert the animation button of each independent variable and their series button.
6. Rename the series button as "Animate clock" and animate it.



Example 2: Exchange of ions in water formation process

The model of this animation is as follows:

In what we can call a first major period, a point M jumps from A to B and from B to C, then coming back to B and to A with a plus sign (+) attached.



Road travelled by M

In the second major period the point M does exactly the same motion but with the minus sign (–) attached.

The abscissa x of M in A, B and C is respectively 0, 1 and 2 as shown in the road travelled by M.

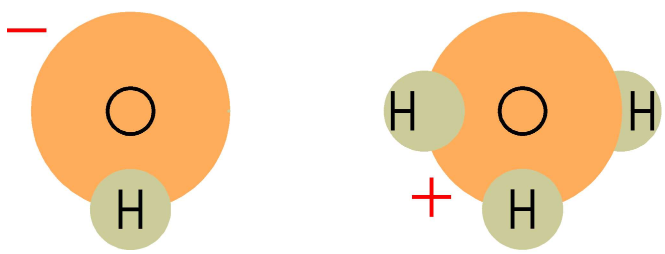
To get the animation under control in terms of timing, just consider that each interval [A, B] and [B, C] is ran in 1 second. Then the two major periods last in total 8 seconds.

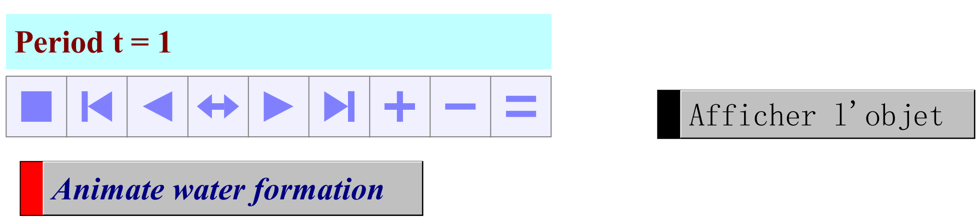
Now the abscissa of M should be expressed in term of time t, that is x (t) and match the values 0, 1 and 2 at A, B and C positions while t is an independent variable with precision 1 and varies in a discrete domain { 1,2,3,4,5,6,7,8 } (because of the jumping situation) .

Then, x (t) =in (t,0, 2) *t+in (t,3, 4) *t%2+in (t,5, 6) *t%4+in (t,7, 8) *t%2

=in (t,0, 2) *t+ (in (t,3, 4) +in (t,7, 8)) *t%2+in (t,5, 6) *t%4

The gain of ion in the first major period can be built upon the function in (t,0, 4) , the non gain of ion and the appearence of the line in the second major period upon in (t,5, 8) and finally the constant appearence of the two hydrogen atoms upon in (t,0, 8) .

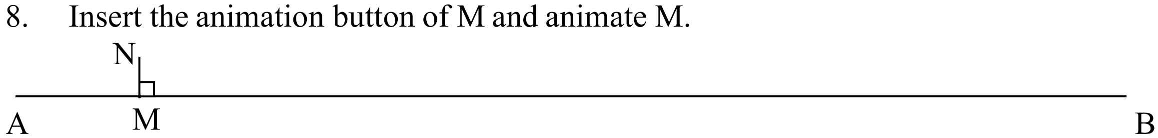




d) The use of data of object in motion

Example: motion of bicycle tire

1. Draw the figure below where AB=15cm and MN=5mm.
2. Draw the circle centered at P passing M.
3. Select A and M and click on the distance tool  to display the distance AM.
4. Select the circle and click on the tool Radius  to display the radius.
5. Define a functional variable -x/y where x is the distance AM and y the radius.
6. Select the circle and define the circle point Q with functional variable taken in rd.
7. Join P and Q , then rotate 120° and 240° segment PQ about P
8. Insert the animation button of M and animate M.



5) Miscellaneous

In general, to start any animation construction you need to have a very clear idea of the various steps of that animation.

There are many ways to get the same result but one could prove to be smarter than another with less construction steps. A good understanding of the use of mathematical functions is really helpful. Here we can advocate that ScienceWord and Class are the unique software offering the most naturel way in applying mathematical functions with all the meaning attached. These two software indeed bring to life the most abstruse concepts in the hand of trained users.

Here we will be studying few examples.

a) Animation of the development of the cube

Expected results

The development of the cube as shown in FigA is a plane geometry object made of six equal squares.

The cube is a 3D object (FigB) with a perpective view where: $\angle CAB=45°$, $\frac{AC}{AB}=0.6$.

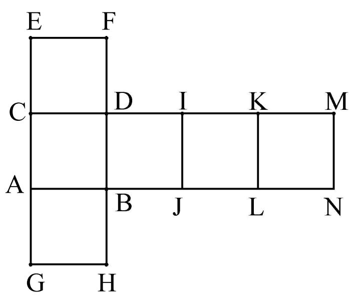


Fig A

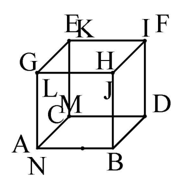
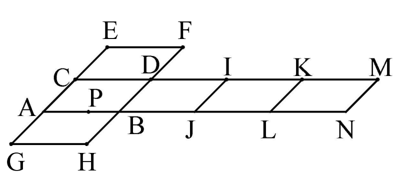
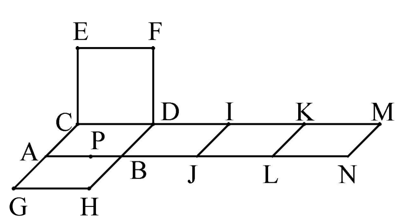


Fig B

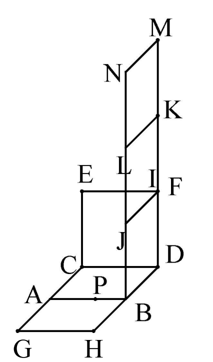
Animation construction steps



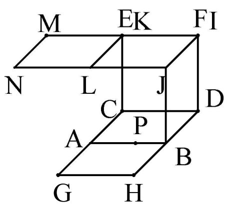
Step 1



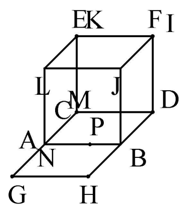
Step 2



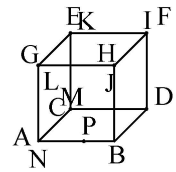
Step 3



Step 4



Step 5



Step 6

Construction 1: Draw ABCD with the 3D view condition (Step 1)

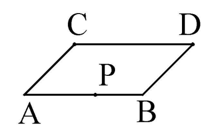
1) Draw a segment AB and define an independent variable T from 0 to 6.

2) We have to construct the point P of the segment [AB] such that when T varies from 0 to 1, the ratio $V_1=\dfrac{AP}{AB}$ varies from1 to 0.6.

Vector From A, to B

$V_1 = 0.60$ $U_1 = 45.00$

T = 1. 00



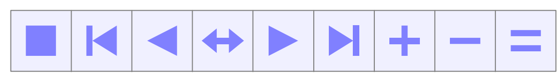


Fig 1

The ratio V_1 is defined as functional variables on [0, 1 [∪ [1,6] as follows :

V_1= (in (x, 0, 1) -in (x, 1, 1)) *(1-0.4*x) +in (x,1,6) *0.6

Then select in this order the ends A and B of the segment and click on " Define an axis abscissa". In the dialog box that opens up check the option variable and select V_1. Click OK to get the point P.

3) The point C is the rotation of the point P around A using the variable angle U_1 that varies from 90° to 45°.

The angle U_1 is defined as functional variables on [0, 1 [∪ [1,6] as follows :

U_1= (in (x,0,1) -in (x,1,1)) * (90- 45*x) +in (x,1,6) *45 where x=T.

Then select in this order the points P and B of the segment and click on " Rotation". In the dialog box that opens up make sure that angle unit is degree, check the option variable and select U_1. Click OK to get the point C.

4) Define the vector $\overrightarrow{AB}$ and translate AC by this vector. Then draw ACDB.

Construction 2: Draw the points E and F (Step 2)

The point E can be obtained by rotating the point A around the point C.

However we have to consider that during the first period that is from 0 to 1s, the points A, C, E are aligned and AC = CE.

During the second period that is from 1s to 2s, the angle ∠ECA increases from 180 to 225° and the ratio $\dfrac{CE}{AB}$ also increases from 0.6 to 1.

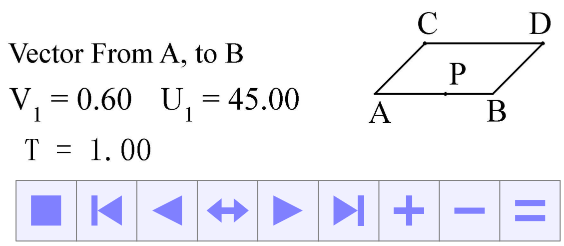


Fig 1

Then angle ∠ECA can be written as a variable U_2 as follows:

U_2= (in (x,0,1) -in (x,1,1)) *180+ (in (x,1,2) -in (x,2,2)) * (180+45* (x-1)) +in (x,2,6) *225, where x is T.

The ratio can be written as a variable V_2 as follows

V_2= in (x,0,1) -in (x,1,1) + (in (x,1,2) -in (x,2,2)) * (1+0.4/0.6* (x-1)) + in (x,2,6) *1/0.6 where x is T.

To construct the point E, you need to construct an auxiliary point E_1 that is the rotation of the point A around C using the variable U_2. Then select in this order the points E_1 and C and click on " Define an axis abscissa". In the dialog box that opens up check the option variable and select V_2. Click OK to get the point E.

Construction 3: Draw the points I and J (Step 3)

The points I and J can be obtained by rotating the point C around the point D and the point A around the point C using the same angle U_3 of rotation defined as follow.

U_3= (in (x,0,2) -in (x,2,2)) *180+ (in (x,2,3) -in (x,3,3)) * (180+ (x-2) *90) +in (x,3, 6) *270. where x is the independent variable T.

Construction 4: Draw the points L and K (Step 4)

The points K and L can be obtained by rotating the point D around the point I and the point B around the point J using the same angle U_4 of rotation defined as follows

U_4= (in (x,0,3) -in (x,3,3)) *180+ (in (x,3,4) -in (x,4,4)) * (180+ (x-3) *90) +in (x,4, 6) *270 where x is the independent variable T.

Construction 5: Draw the points M and N (Step 5)

The points M and N can be obtained by rotating the point I around the point K and the point J around the point L using the same angle U_5 of rotation defined as follows

U_5= (in (x,0,4) -in (x,4,4)) *180+ (in (x,4,5) -in (x,5,5)) * (180+ (x-4) *90) +in (x,5, 6) *270

Construction 6: Draw the points G and H (Step 6)

During the last period of rotation, the segments AG and BH sizes increase from 0.6 × AB to AB. Then first we need to consider the construction of the auxliriary points G_1 and H_1 using the same ratio V_3 defined as follows:

V_3= 1+in (x,0,5) -in (x,5,5) +in (x,5,6) * (1+0.4/0.6* (x-5))

Select in this order C and A then click on " Define an axis abscissa". In the box that opens up check the option variable and select the variable V_3.; the result is a point G_1.

Select in this order D and B and click on " Define an axis abscissa". In the box that opens up check the option variable and select the variable V_3.; the result is a point H_1.

The points G and H are obtained by rotating the point G_1 around the point A and the point H_1 around the point B using the same angle U_6 , defined as follows:

U_6=in (x,5,6) * (x-5) * (-135) , where x is the variable T.

The final result is shown below

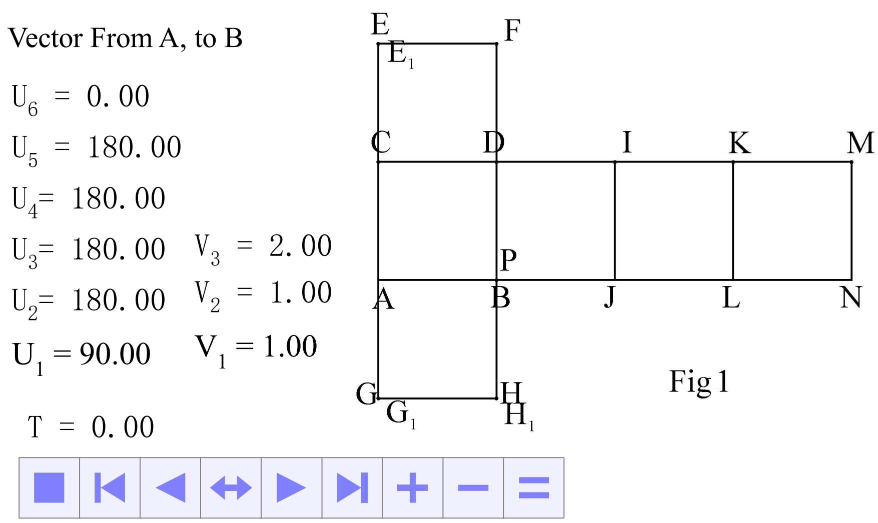


Fig 1

b) Animation of a point on a triangle with a speed proportional to the side run while the whole triangle is independently of its size run within a fixed period

Step1:

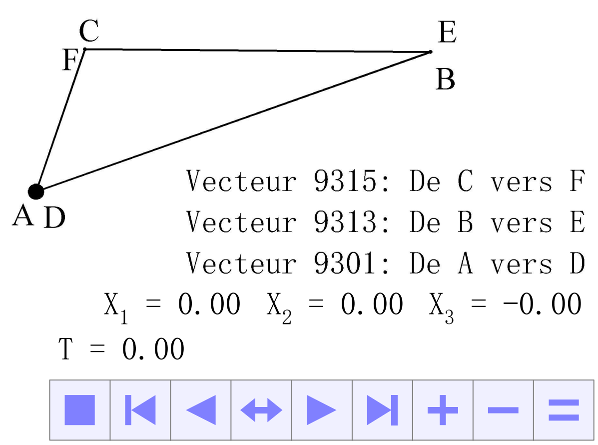
Draw a triangle ABC and define an independent variable T from 0 to 3..

Define the functional variables:

X_1=(in (x,0,1) -in (x,1,1)) *x+in (x,1,3) , where X=T

X_2=(in (x,1,2) -in (x,2,2)) * (x-1) +in (x,2,3) , where X=T

X_3=in (x,2,3) * (x-2) , where X=T



Step2:

Set T=0.2 and construct the point D having X_1 as abscissa in (A, B) coordinates. Then define the vector A to D. and hide the point D.

Set T=1.2 and construct the point E having X_2 as abscissa in (B, C) coordinates. Then define the vector B to E and hide the point E.

Set T=2.2 and construct the point F having X_3 as abscissa in (C, A) coordinates. Then define the vector C to F and hide the point F.

Step3:

Now you have to translate the point A by an ordered sum of the three vectors defined.

To do so, select the point and apply the translation of vector A to D; then apply to the new point obtained the translation of vector B to E; then apply to the point obtained the translation of vector C to F. Then access the properties dialog box of the last point obtained; uncheck keep the original object style and set 0.7mm as the point size.

Note:

You can modify at will, the duration of the animation. For example, you may set from the independent variable T dialog box a rate of 1 unit/3s to get a longer duration of animation. However the modification of the size of the triangle does not affect the duration of the animation.

This method can be generalized to any n sided polygon. You may have to define the independent variable T from 0 to n.

c) Animation of a triangle point with constant speed while running the whole triangle within a fixed period.

Step 1

Draw a triangle ABC and define an independent variable T from 0 to3, with a speed 1unit/second.

Display the side of each side AB, BC, CA of the triangle and its perimeter L.

Define the functional variables AB/L, BC/L, CA/L, (AB+BC) /L.

Define the thre folowing functional variables:

X_1= (in (x,0,y) -in (x,y,y)) *x/y+in (x,y,1) , where x=T, y =AB/L

X_2= (in (x, y, z) -in (x, z, z)) * (x-y) /a+in (x, z, 1) , where x=T, y =AB/L, z= (AB+BC) /L, a=BC/L.

X_3=in (x,y,1) * (x-y) /z, où x=T, y = (AB+BC) /L, z=CA/L.

Step 2

Set T=0.1 (value betwen 0 and AB/L) and construct the point D having X_1 as abscissa in (A, B) coordinates. Then define the vector A to D. and hide the point D.

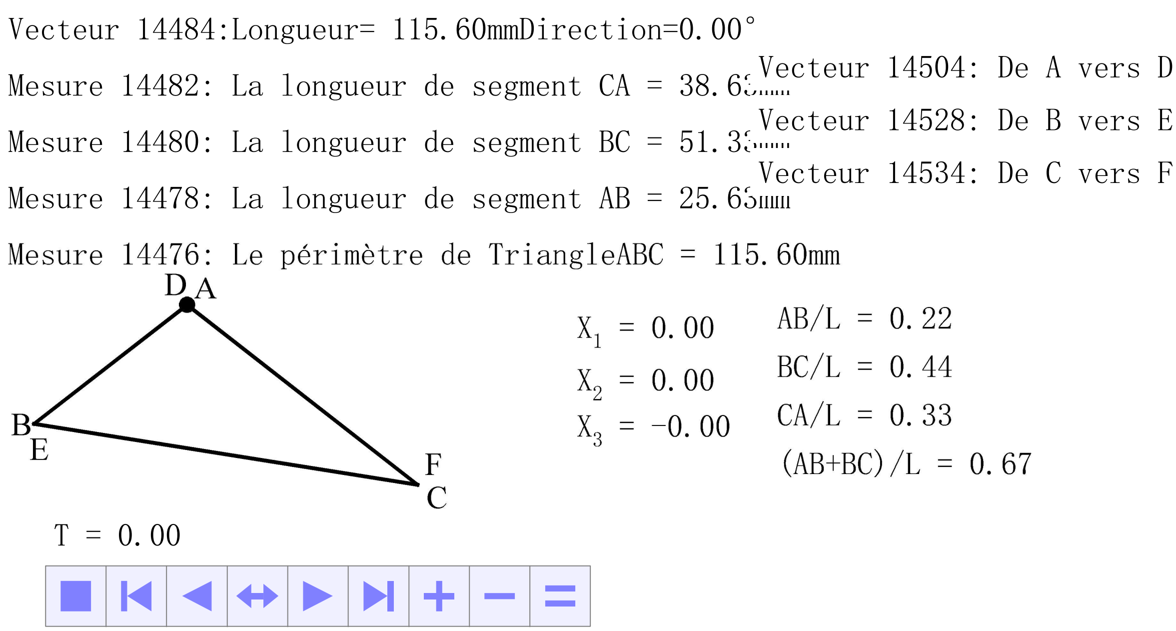
Set T=0.4 (value betwen AB/L and (AB+BC) /L) and construct the point E having X_2

as abscissa in (B, C) coordinates. Then define the vector B to E and hide the point E.

Set T=0..8 (value betwen (AB+BC) /L and 1) and construct the point F having X_3 as abscissa in (C, A) coordinates. Then define the vector C to F and hide the point F.

Step3:

Now you have to translate the point A by an ordered sum of the three vectors defined. To do so, select the point and apply the translation of vector A to D; then apply to the new point obtained the translation of vector B to E; then apply to the point obtained the translation of vector C to F. Then access the properties dialog box of the last point obtained; uncheck keep the original object style and set 1 mm as the point size.



Note:

You can modify at will, the duration of the animation. For example, you may set from the independent variable T dialog box a rate of 1 unit/3s to get a shorter duration of animation. However the modification of the size of the triangle does not affect the duration of the animation.

This method can be generalized to any n sided polygon.

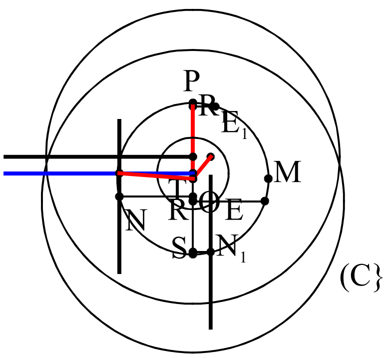
d) 3D rotation with double axis of rotation

1. Draw a circle of 10mm radius with a centre O, and the point P on the circle of polar angle 90°.
2. Draw the point M of the circle with having 0° as polar angle value.
3. Select a point E of the circle (O, OP) , then draw its projection R on (OP) and draw any circle (C) centered at R. Insert the animation button of the point E.
4. Rotate E around O about 90° to get a point E_1 , then project E_1 on (OP) to get a point R_1 and draw OR_1 in red .

5. Select a point N of the circle (O, OP) and draw its projection T on (OP). Insert the animation button of the point N.
6. Rotate N around O about 90° to get a point N_1 and project N_1 on (OP) to get a point T_1.
7. Draw the image of (C) by enlargement of $\frac{OT}{OP}\sin(\angle MOT)$ and centre O. Then draw the intersection of the perpendicular to OP passing the centre of the circle image and the parallel to OP passing N. Finally draw in red the segment that links this intersection point to O.
8. Draw the image of (C) by enlargement of $\frac{OT_1}{OP}\sin(\angle MOT_1)$ and centre O. Then draw the intersection of the perpendicular to OP passing the centre of the circle image and the parallel to OP passing N_1. Finally draw in red the segment that links this intersection point to O.
9. Animate the points E, then animate the point N.

Note: ∠MOT and ∠MOS are positive and negative angle value.

The expected result is shown as follows



(OT/OP)sin(MOT) = -0.24

(OS/OP)sin(MOS) = -0.97

Mesure 17067: Angle MOT = -90.00°

Mesure 17065: Angle MOS = -90.00°

Vecteur 16993:Longueur= 10.00mmDirection=90.00°

OT = 2.36mm

OP = 10.00mm

OS = 9.72mm

Animate point E

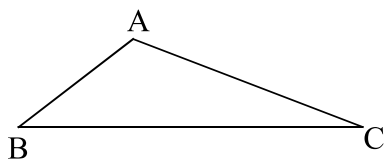
Animate point N

6) Note on calculation using functional variables

During the computation of expressions including data like length, area, angle, the conversion is automatically done in mm, mm^2 and radians as noted in the dialogue box.

Example 1:

Let draw a triangle ABC and display its various internal angles values in degree, rd and πrd , the area in inch square and the perimeter in centimeter as follows.



Area of TriangleABC = 0.41inch²

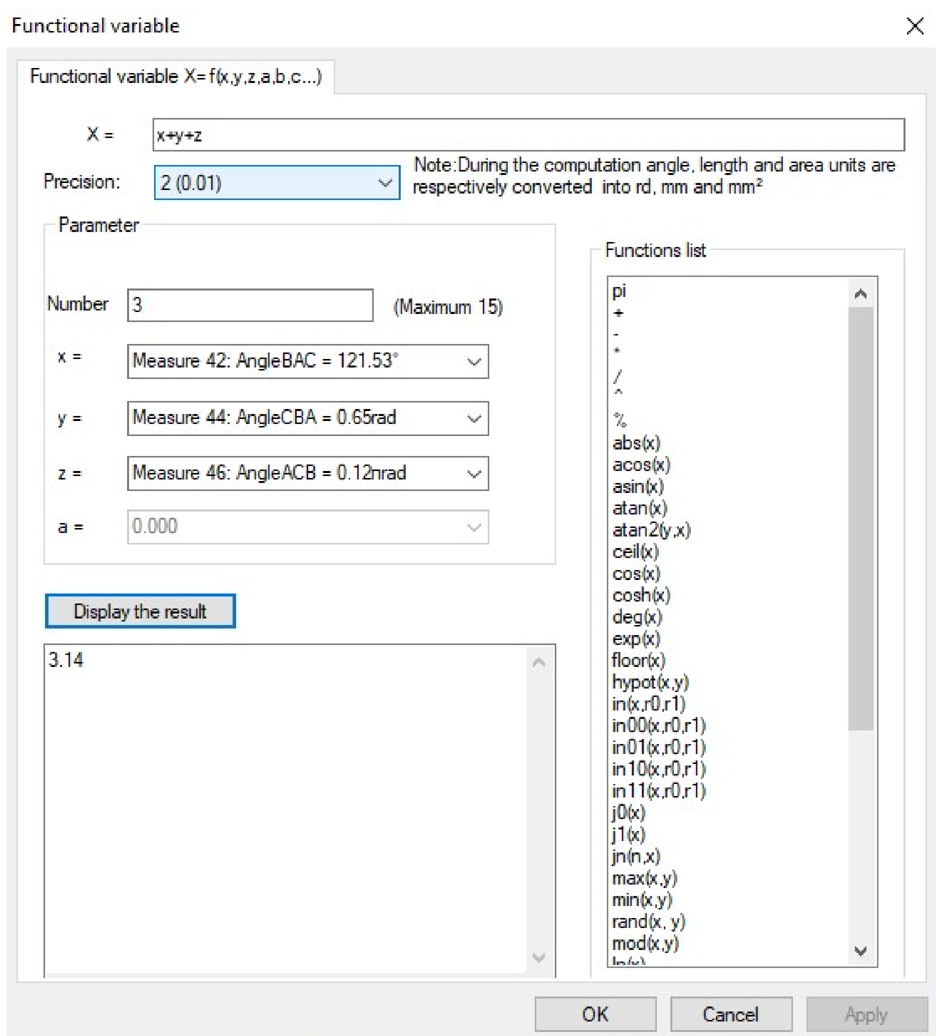
Perimeter of TriangleABC = 9.71cm

Measure 42: AngleBAC = 121.53°

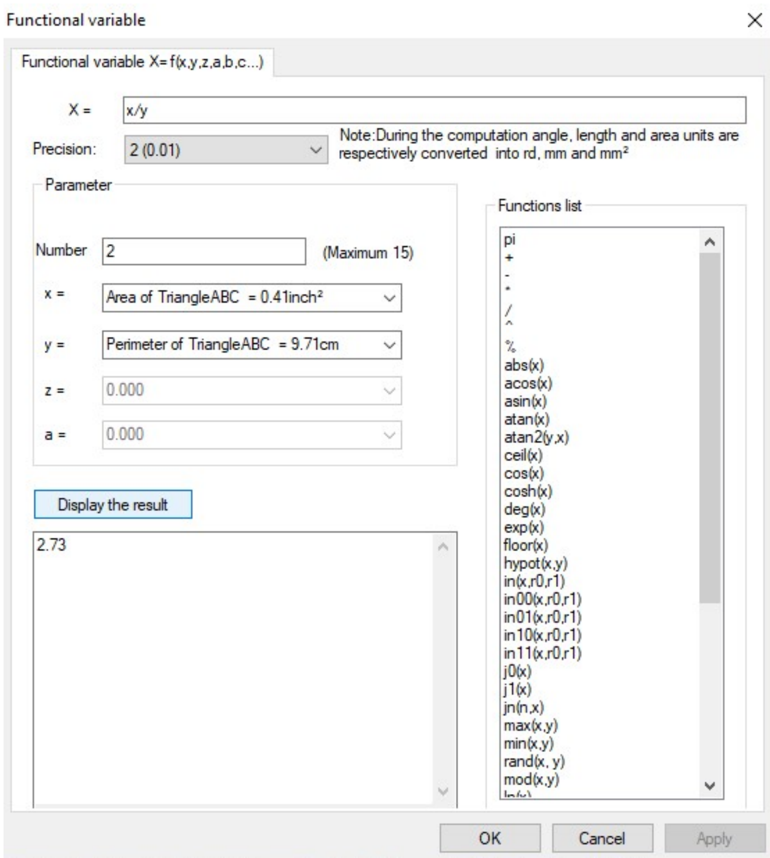
Measure 44: AngleCBA = 0.65rad

Measure 46: AngleACB = 0.12πrad

The following image shows the calculation in the functional dialog box:
X=x+y+z, where x=Measure 42: ∠BAC, y=Measure 43: ∠CBA, z=Measure 46: ∠ACB .
When you click on the "Display the result" button, you get 3. 14 because during the calculation, the conversion of each angle measure is done in radian.
To get the result in degree, you just have to apply the "deg" function available in the function list. That is deg (x+y+z) =deg (3.14) =180.



The image below shows another calculation in the functional dialog box:

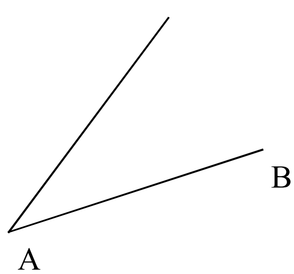


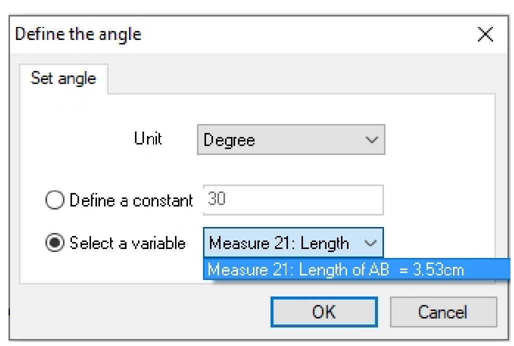
X=x/y , where x=Area of TriangleABC and y= Perimeter of Triangle ABC
During the calculation the area has been converted in mm^2 and the perimeter in mm.

Example 2:

The same result is obtained when applying a tranformation using the same kind of variables with different meaning.
For example the image below shows the rotation of the line AB around the point A using its length 3.53cm as angle with the degree option.

Measure 21: Length of AB = 3.53cm

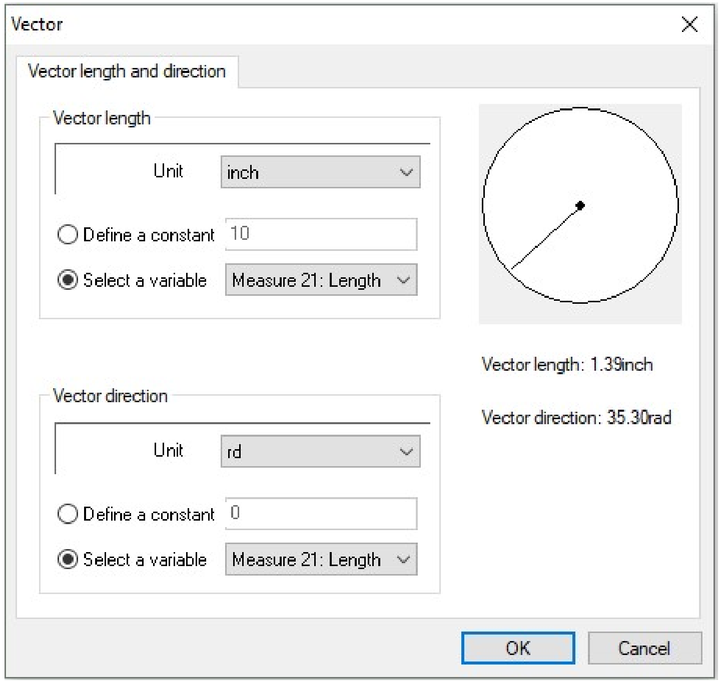




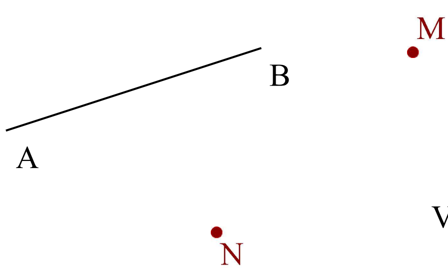
This result is in fact a rotation of 35.3 degrees around A because during the operation, the conversion of the length has been done in mm.

Example 3

If you are defining a vector using AB length (3.53cm) and inch as unit, the result will not be displayed in mm. This conversion will be done in inch appropriately, that is 1.39 inch, as you are using the length AB for a variable of same nature "length".



But if you are chosing the same AB length (3.53cm) as direction, then as the nature of the use of AB length changes, the conversion is done in mm, that is 35.3mm with the unit chosen (rd) that is 35.3 rd as illustrated below. In this drawing, the point M has been translated by the vector defined and the result is the point N.



Measure 21: Length of AB = 3.53cm

Vector 9763:Length=1.39inch, direction=35.30rad

8) A concept for the creation of periodic functions

Let consider a real parameter m ($m \neq 0$) and the function $f_m(x) = x - m\left\lfloor \frac{x}{m} \right\rfloor$, having $\mathbb{R}$ as domain. Then $f_m(x)$ is a periodical function with périod $\mathrm{T} = |m|$ and we have:

- If $m > 0$, then $f_m(\mathbb{R}) = [\,0, m\,[$;

In particular, $\forall n \in \mathbb{N}^*,\ f_m([0, nm]) = [0, m[$.

■ If $m < 0$, then $f_m(\mathbb{R}) =]m, 0]$.

In particular, $\forall n \in \mathbb{N}^*,\ f_m([nm, 0]) =]m, 0]$.

This concept has practical applications in complex dynamic geometric constructions.

Application

In the drawing opposite, the independent variable t is defined from 0 to 6; "**Animate the object t**" is the animation button of the independent variable.

The functional variables ***u*** and *v* are defined as follow:

u = x-1.5*floor (x/1.5) , where x= t (t is the independent variable) ;

v = in (x,0,0.75) *x+in01 (x,0.75,1.50) * (1.50 - x) ,

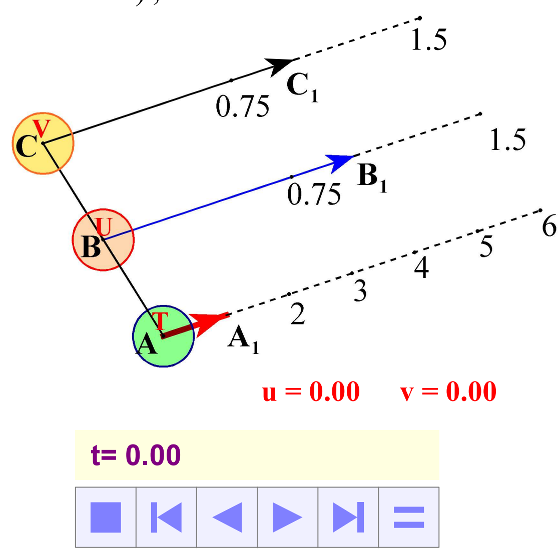
où x= u.

In (A, A_1) coordinates, the abscissa of the centre T of the green circle is t.

In (B, B_1), coordinates, the abscissa of the centre U of the red circle is u.

In (C, C_1), coordinates, the abscissa of the centre V of the yellow circle is v.

Upon clicking on the animation button "Animate the object t", you can note that the movement of the red circle of center U and that of the yellow circle of centre V are periodic of period $T_1 = 1.5$; The movement of the green circle is periodic of period $T_2 = 6$.



7) Conclusion

Animation covers various aspects of the real life. More of this is that animation brings ideas to real forms and clarifies concepts.

Animation comes under control when it is linked to timing. A group of animations can be linked to a period or a time t.

We can compare various activities under the time t to a n-uplet, a kind of relation defined as follows: $t(t_1, t_2, \cdots t_n) \longrightarrow (x_1(t_1), x_2(t_2), \cdots, x_n(t_n))$ where $x_i(t_i)$ is an activity or a group of activities within a period or at a time t_i.

It has been shown in 4) how to deal with all kind of variables domains.

Animation proves to be very useful in drawing. One of the traditional applications is the locus that displays at once a sample of object in various positions.
Other significant results are obtained when animation comes under data control. Then the behaviour of the object in motion can be shown at a particular point, time or for a certain datum. The user can duplicate the object through the actions copy and paste while considering different data. This can be applied when drawing the process of a whole complex system.

Many practical tutorials are available in Class file. the reader may check from our website www.scienceoffice.com
or send an Email to the following address ecomlan@scienceoffice.com

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